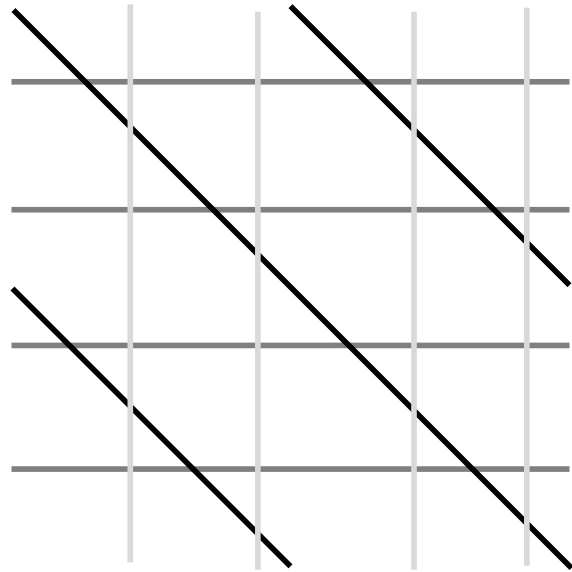




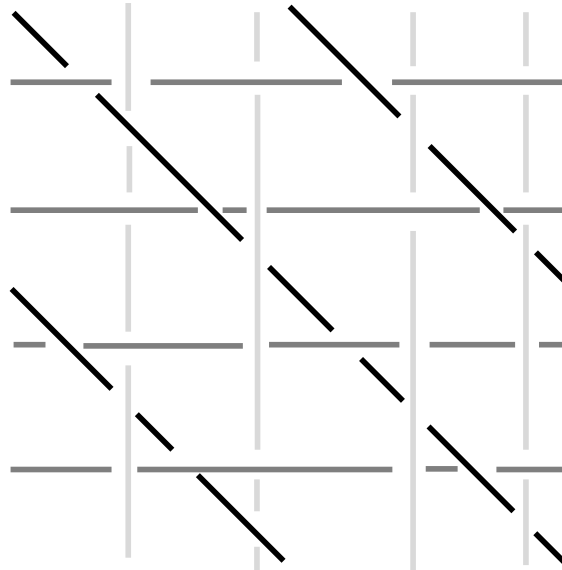
Periodic Weaving Diagrams

Structures on Surfaces
CIRM

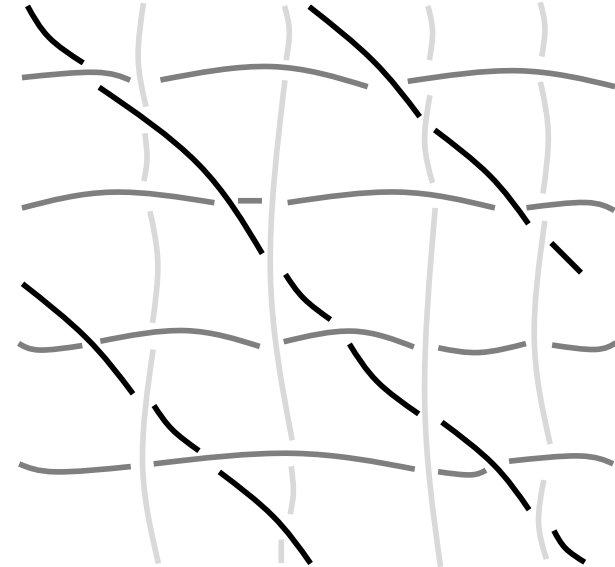
Definition of an Untwisted Weave



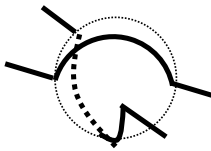
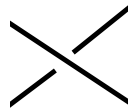
(A)



(B)



(C)



\mathbb{E}^2

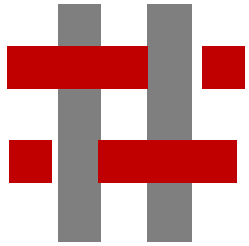
\mathbb{X}^3

(A) Sets of geodesics belonging to different color groups in the Euclidean plane.

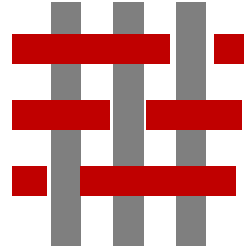
(B) Crossing information to each intersection in the Euclidean plane.

(C) Untwisted weave in the thickened Euclidean plane.

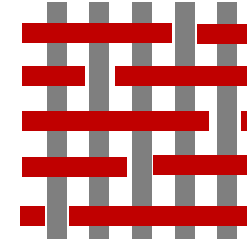
Examples of Crossing Sequences



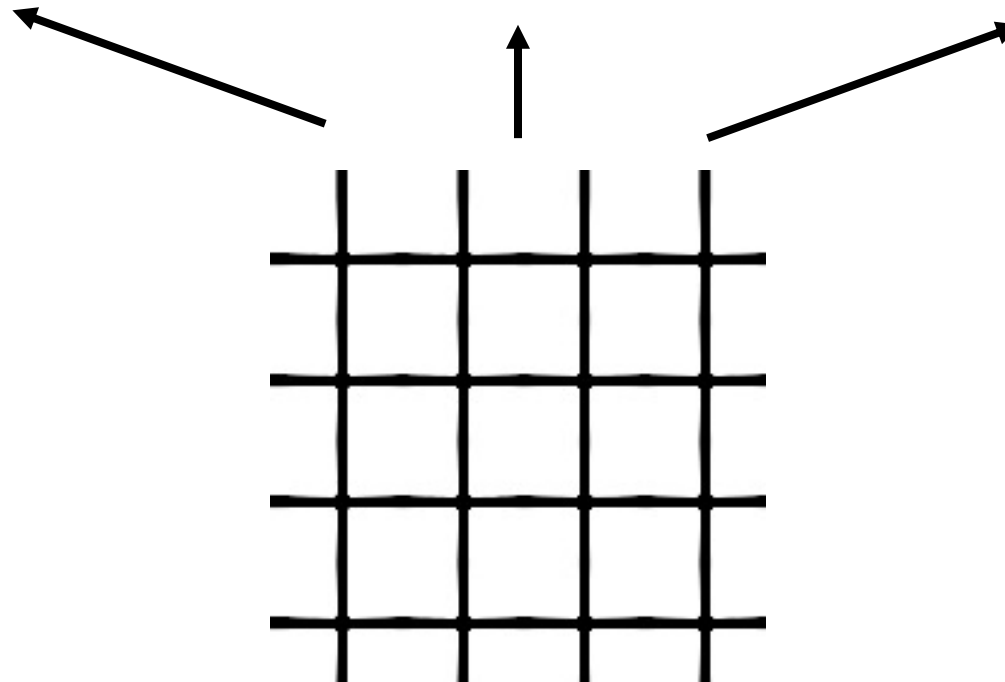
$$C_{\text{red/grey}} = (+1, -1)$$



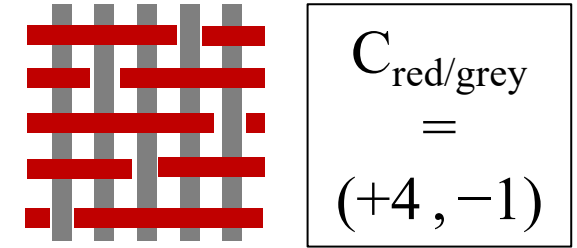
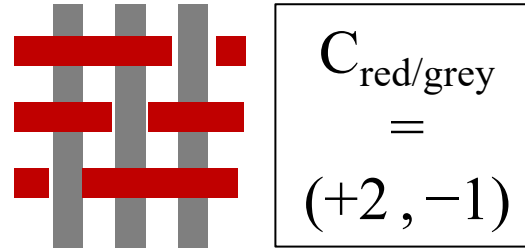
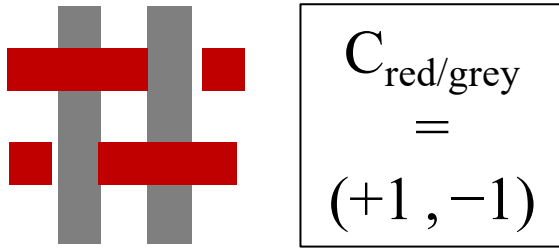
$$C_{\text{red/grey}} = (+2, -1)$$



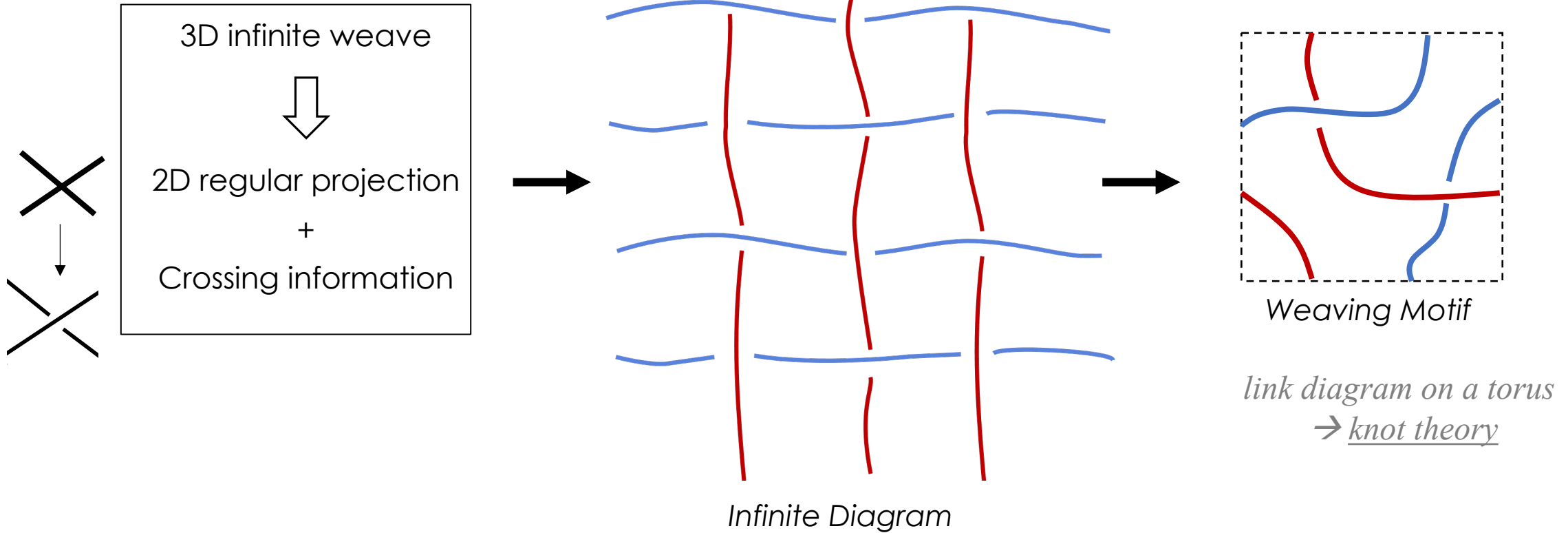
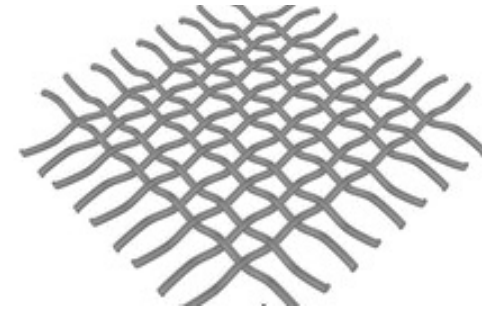
$$C_{\text{red/grey}} = (+4, -1)$$



Examples of Crossing Sequences

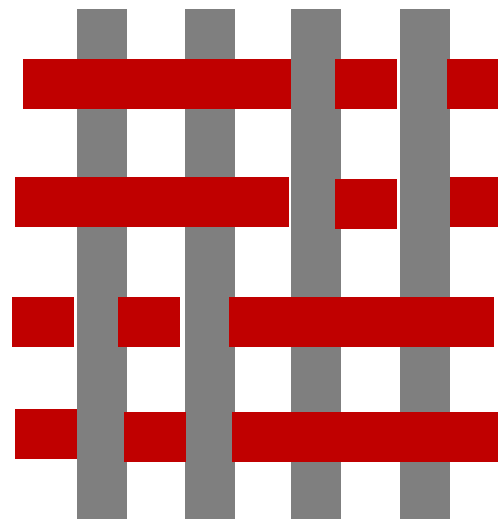


Definition of a Weaving Diagram

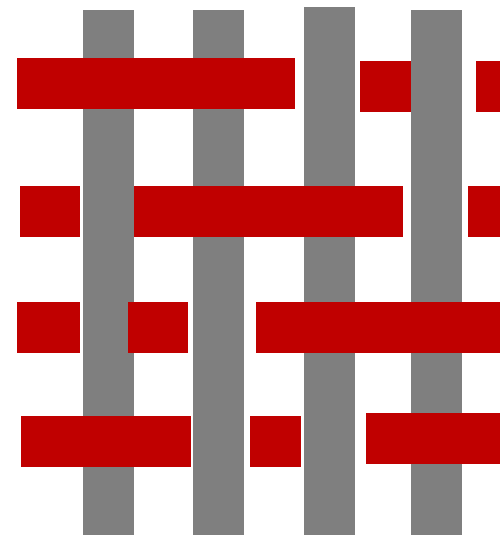
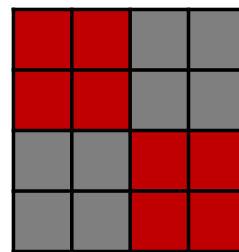


Equivalence Classes of Weaves

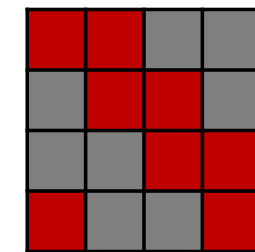
Problem: different weaving diagrams can be characterized by the same pair (**graph, crossing information**).
→ We must therefore find a way to distinguish them.



Biaxial Basket
(+2, -2)



Biaxial Twill
(+2, -2)

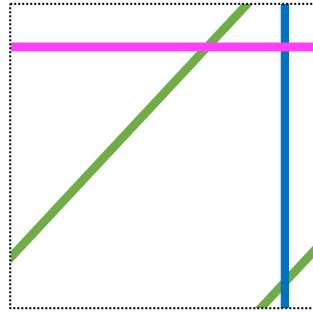


Same number of sets of threads and same set of crossing sequences
BUT different “designs”.

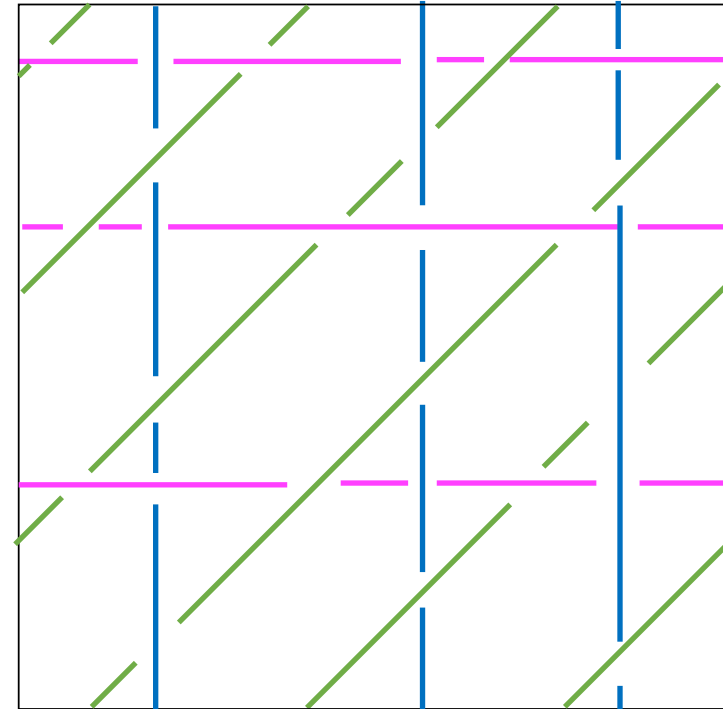
NB: different properties for the materials

- *Basket weave: reversible, rigid and strong*
- *Twill weave: non-reversible, textured and flexible*

Example: Weaving Motif \rightarrow Crossing Matrices



- **Number of sets of threads:** $N = 3$
- Γ : sets of geodesics $(1,0)$, $(0,1)$, $(1,1)$
- **Set of crossing sequences:** $\Sigma = \{(2,1)_3\}$
- **Set of crossing-matrices Π :**



$M_{1,2}$

=

+1	+1	-1
-1	+1	+1
+1	-1	+1

$M_{2,3}$

=

+1	-1	+1
+1	+1	-1
-1	+1	+1

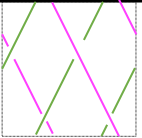
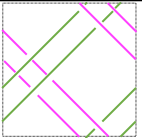
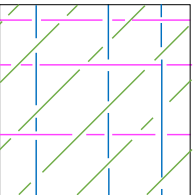
$M_{3,1}$

=

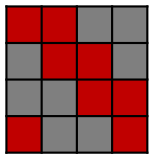
+1	+1	-1
+1	-1	+1
-1	+1	+1

Achievement Goal

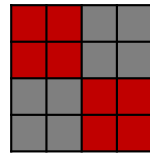
Classification table of weaves with relevant parameters for applications

CLASSIFICATION SQUARE WEAVING DIAGRAMS: N = 2							
Name	Set of Crossing Sequences	Crossing number (Writhe)	Minimal Diagram	Set of Crossing Matrices	Matrices	Symmetry	...
Twill Square Weave (2,2)	{(2,2)}	4 (0)		$\left\{ \begin{pmatrix} +1 & +1 & -1 & -1 \\ -1 & +1 & +1 & -1 \\ -1 & -1 & +1 & +1 \\ +1 & -1 & -1 & +1 \end{pmatrix} \right\}$	Rank = 2 (Diagonal configuration)	?	...
Basket Square Weave (2,2)	{(2,2)}	8 (0)		$\left\{ \begin{pmatrix} +1 & +1 & -1 & -1 \\ +1 & +1 & -1 & -1 \\ -1 & -1 & +1 & +1 \\ -1 & -1 & +1 & +1 \end{pmatrix} \right\}$	Rank = 1 (Plain configuration)	?	...
CLASSIFICATION KAGOME WEAVING DIAGRAMS: N = 3							
Kagome Weave (2,1) ₃	{(2,1), (2,1), (2,1)}	27 (3)		$\left\{ \begin{pmatrix} +1 & +1 & -1 \\ -1 & +1 & +1 \\ +1 & -1 & +1 \\ \vdots & \vdots & \vdots \\ +1 & +1 & -1 \\ +1 & -1 & +1 \\ -1 & +1 & +1 \\ \vdots & \vdots & \vdots \\ +1 & -1 & +1 \\ +1 & +1 & -1 \\ -1 & +1 & +1 \end{pmatrix} \right\}$	Rank = 3 Rank = 3 Rank = 3 (Diagonal configuration)	?	...

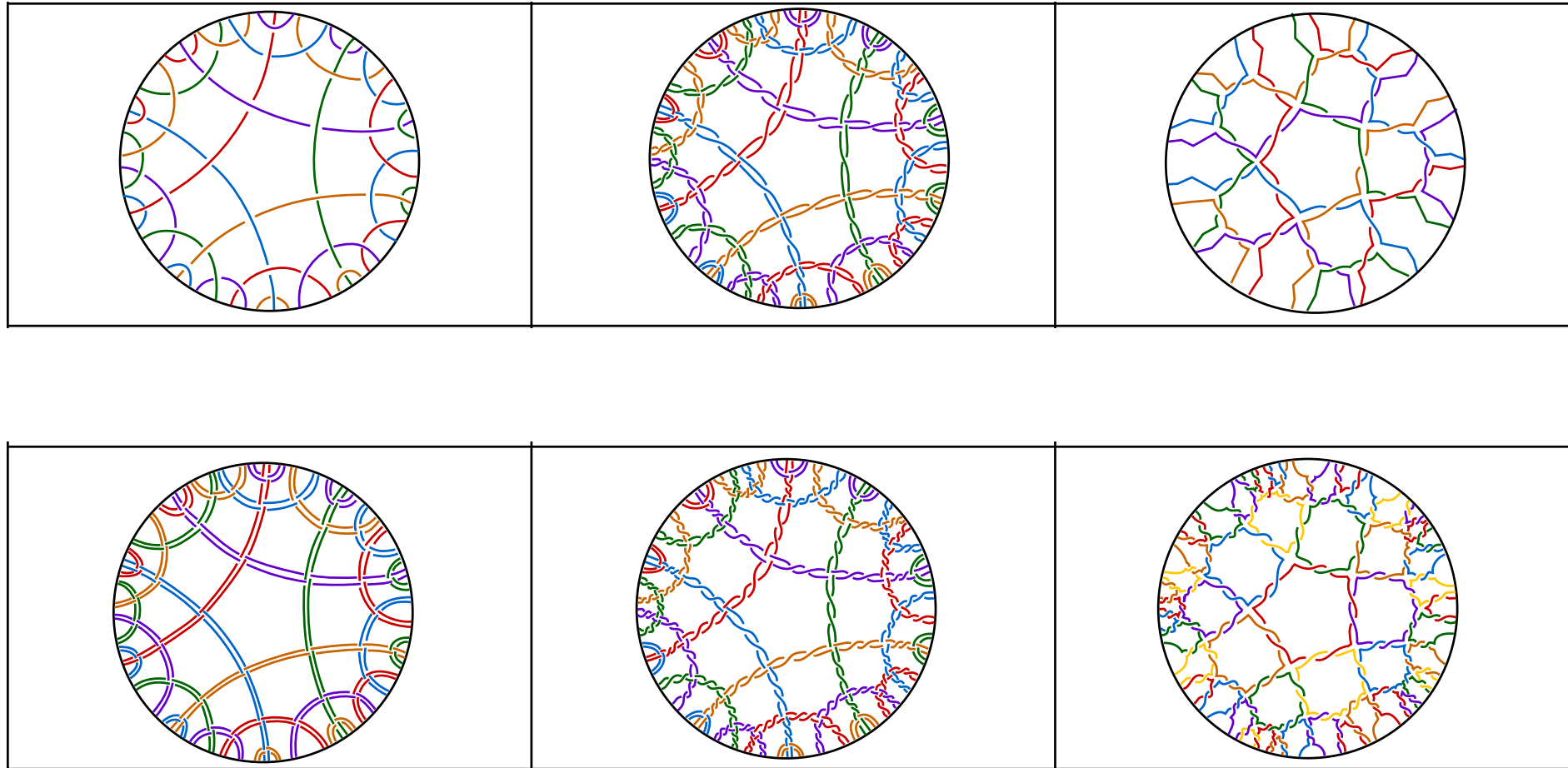
Biaxial Twill
(+2, -2)

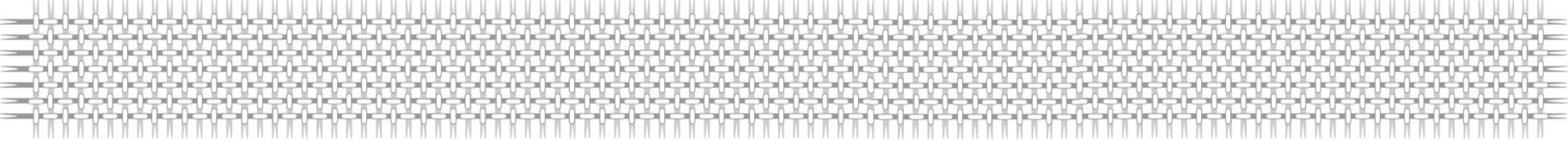


Biaxial Basket
(+2, -2)



Hyperbolic Case ?





THANK YOU VERY MUCH
FOR YOUR ATTENTION

Q&A

This work is supported by a Research Fellowship from JST CREST Grant Number JPMJCR17J4

Ref: M. Fukuda, M. Kotani, S. Mahmoudi. Classification of doubly periodic untwisted (p,q) -weaves by their crossing number, **arXiv:2108.09464**