Computing Periodic Points on Veech Surfaces

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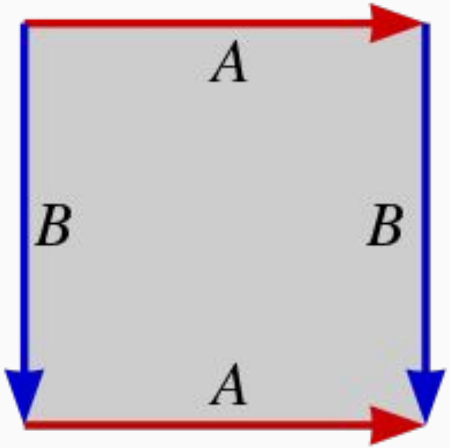
CIRM Structures on Surfaces — May 2nd 2022
Joint with: Zawad Chowdhury, Samuel Everett, Destine Lee

Translation Surfaces

"Polygons + paired opposite sides, up to cut/paste"



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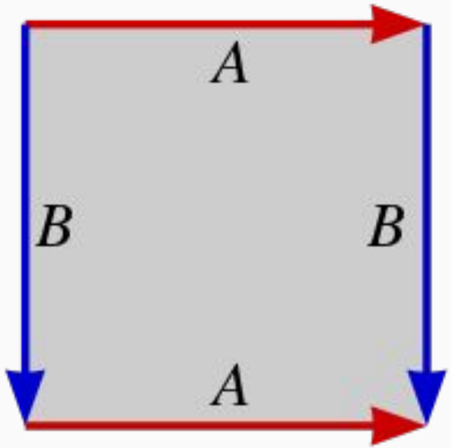
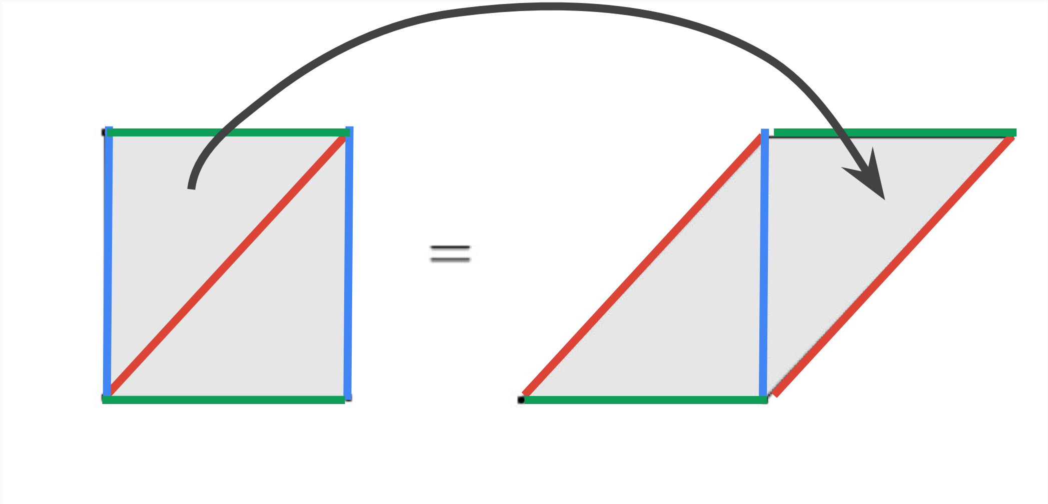
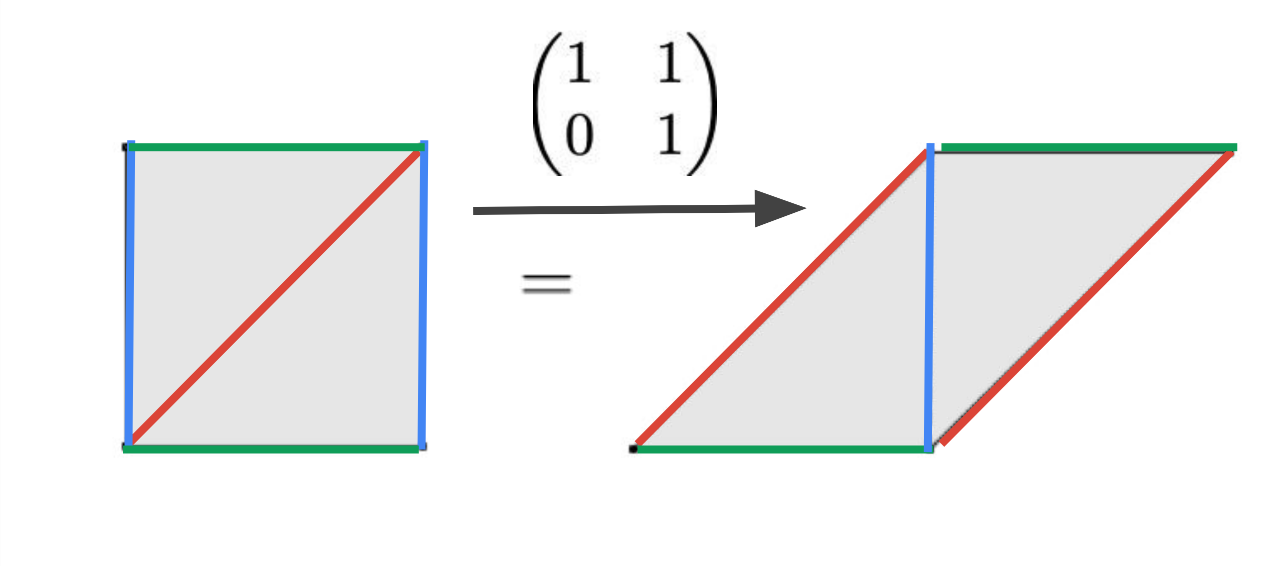
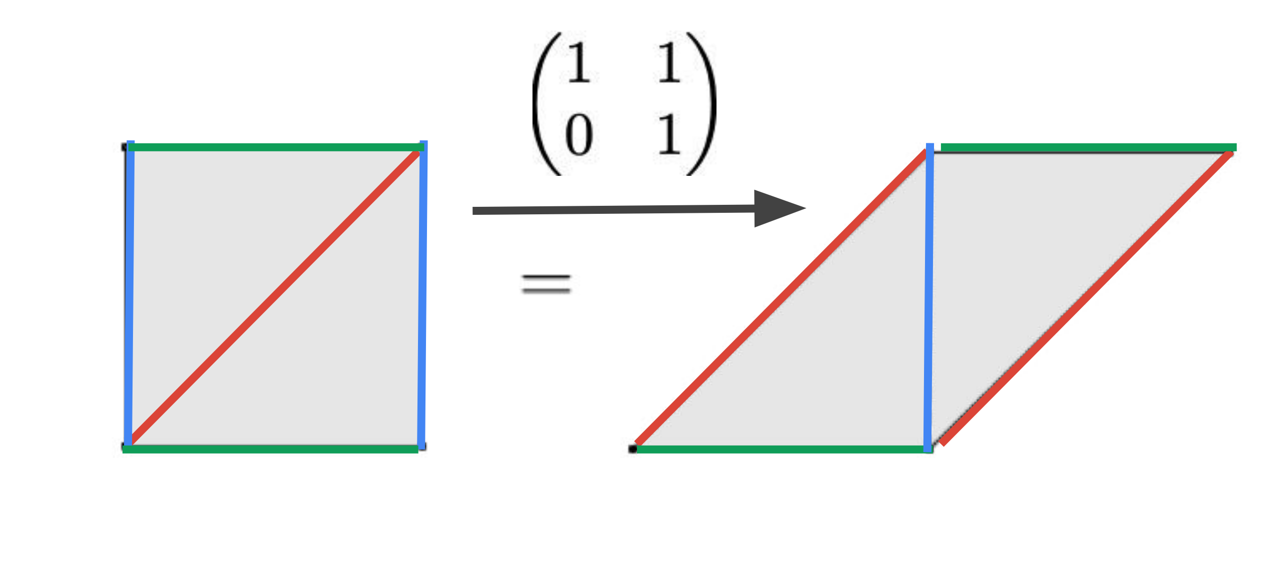




Can also apply matrices...



...to form a "cut/paste group"



$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \in \mathrm{SL}(M)$$

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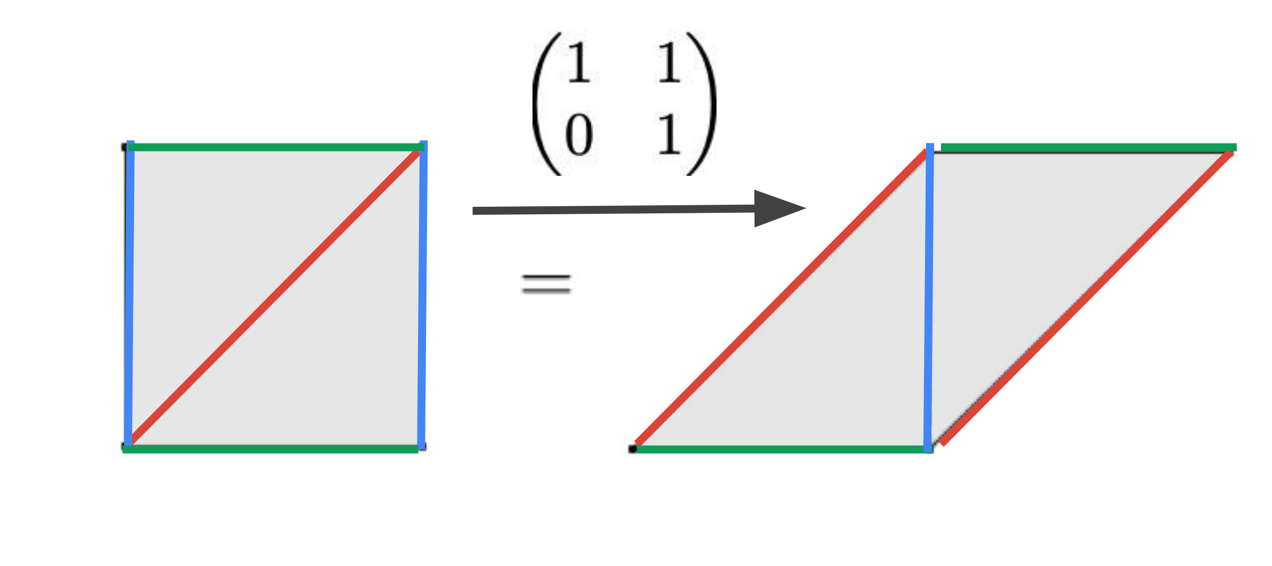

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$$\mathrm{SL}(T^2) \cong \mathrm{SL}(2,\mathbb{Z})$$

Veech surfaces

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we say that M is a **Veech surface**.

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We'll restrict to Veech surfaces in what follows.

Think: Dynamically similar to torus

Periodic Points

A point in M is **periodic** if it has finite SL(M)-orbit

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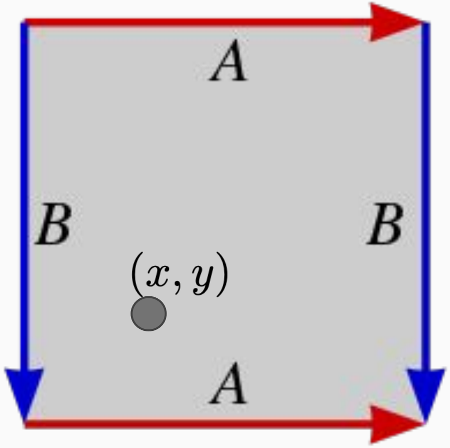

$$(x, y) \text{ periodic} \iff x, y \in \mathbb{Q}$$

Periodic Points

(Gutkin-Hubert-Schmidt '03):
If M is Veech and "not torus-like",
then it has **finitely many** periodic points.

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Applications to billiards problems,
evidence for orbit closure,
counting holomorphic sections…

Previous Classifications

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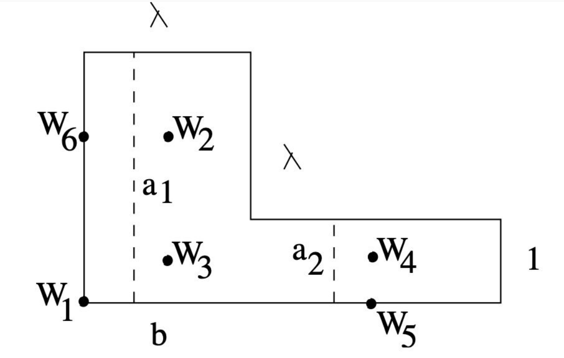
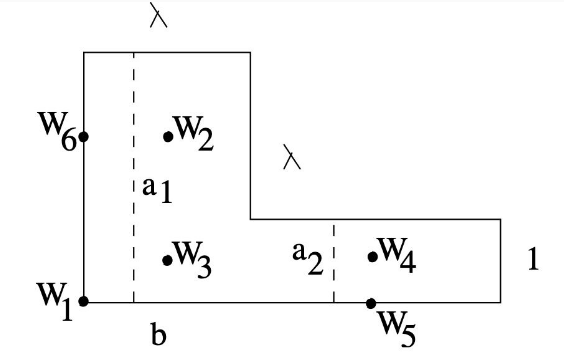
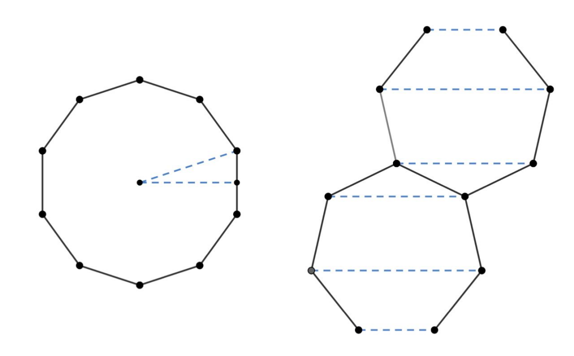


Möller:

Genus 2

Previous Classifications





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Genus 2

Apisa-Saavedra-Zhang:
Double n-gons,
Regular 2n-gons

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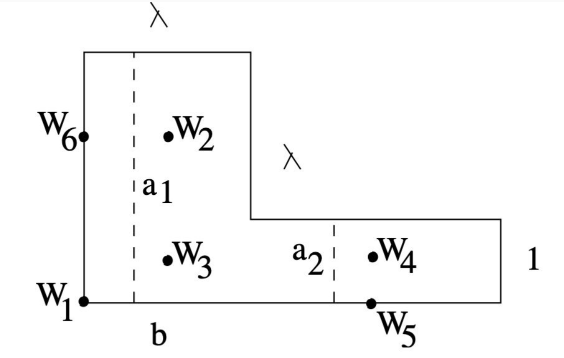

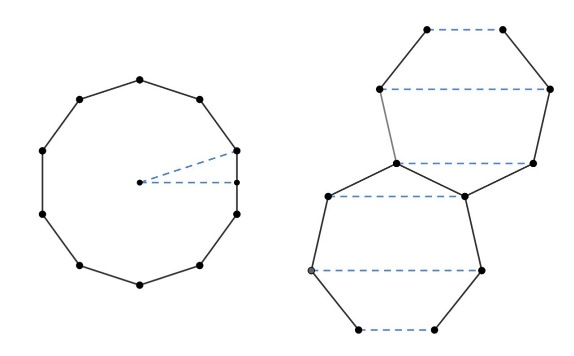

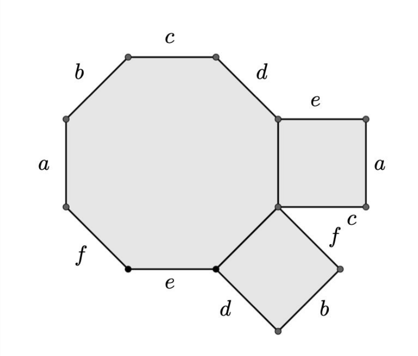

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Genus 2

Apisa-Saavedra-Zhang:
Double n-gons,
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Wright:
Ward-Veech

How can we classify periodic points on general Veech surfaces?

An Algorithm

Theorem (CEFL ‘21): There is an algorithm that, given a “non-torus-like” Veech surface as input, outputs the periodic points on that surface.

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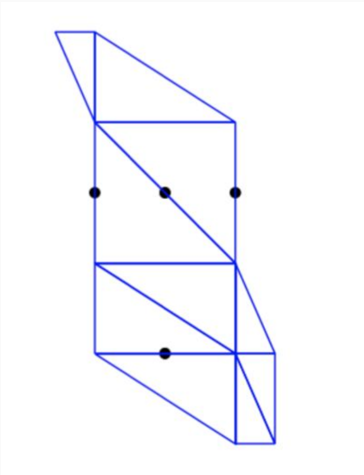
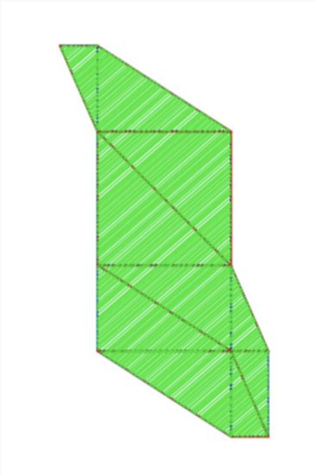
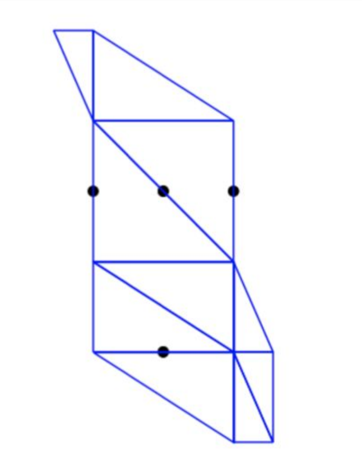


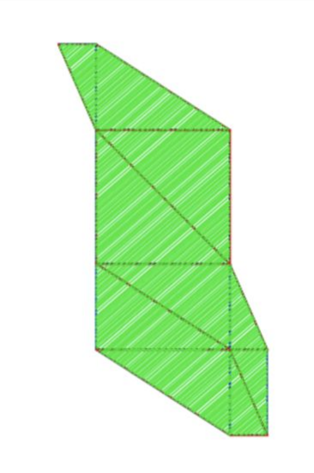


$$\begin{pmatrix} -2\sqrt{17}-9 & \frac{3\sqrt{17}}{2}+\frac{17}{2} \\ \frac{-7\sqrt{17}}{4}-\frac{31}{4} & \frac{3\sqrt{17}}{2}+\frac{13}{2} \end{pmatrix}$$

Applications

Theorem (CEFL ‘21): Prym eigenforms in the minimal stratum in genus 3 with discriminant at most 100 have only fixed points of the Prym involution as periodic points.





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Theorem (F, in progress): Prym eigenforms in the minimal strata in genera 2, 3 and 4 have only fixed points of the Prym involution as periodic points.

Thanks!

Arxiv: 2112.02698

Github: sfreedman67/samsurf