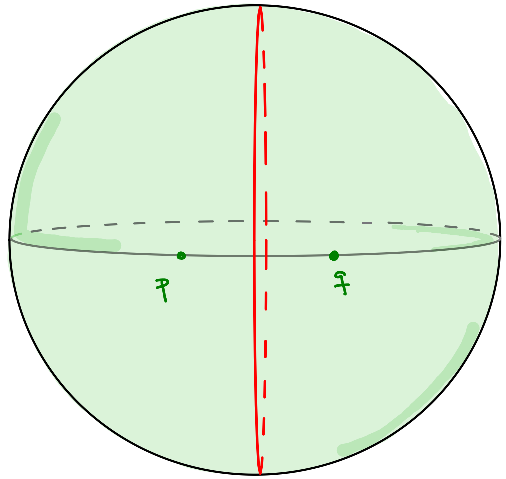
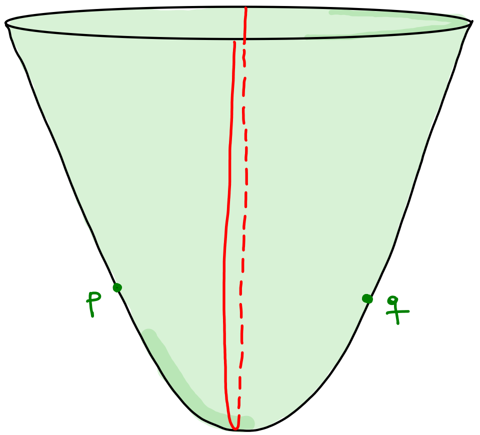
The geometry of thickened bisectors

Hana Dal Poz Kouřimská (IST Austria)

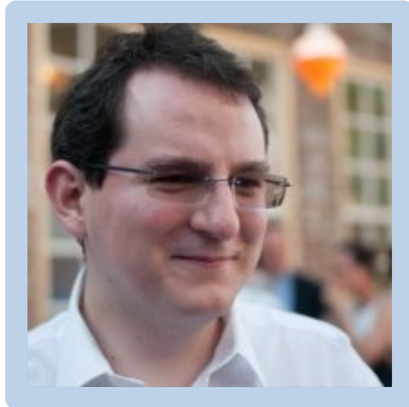


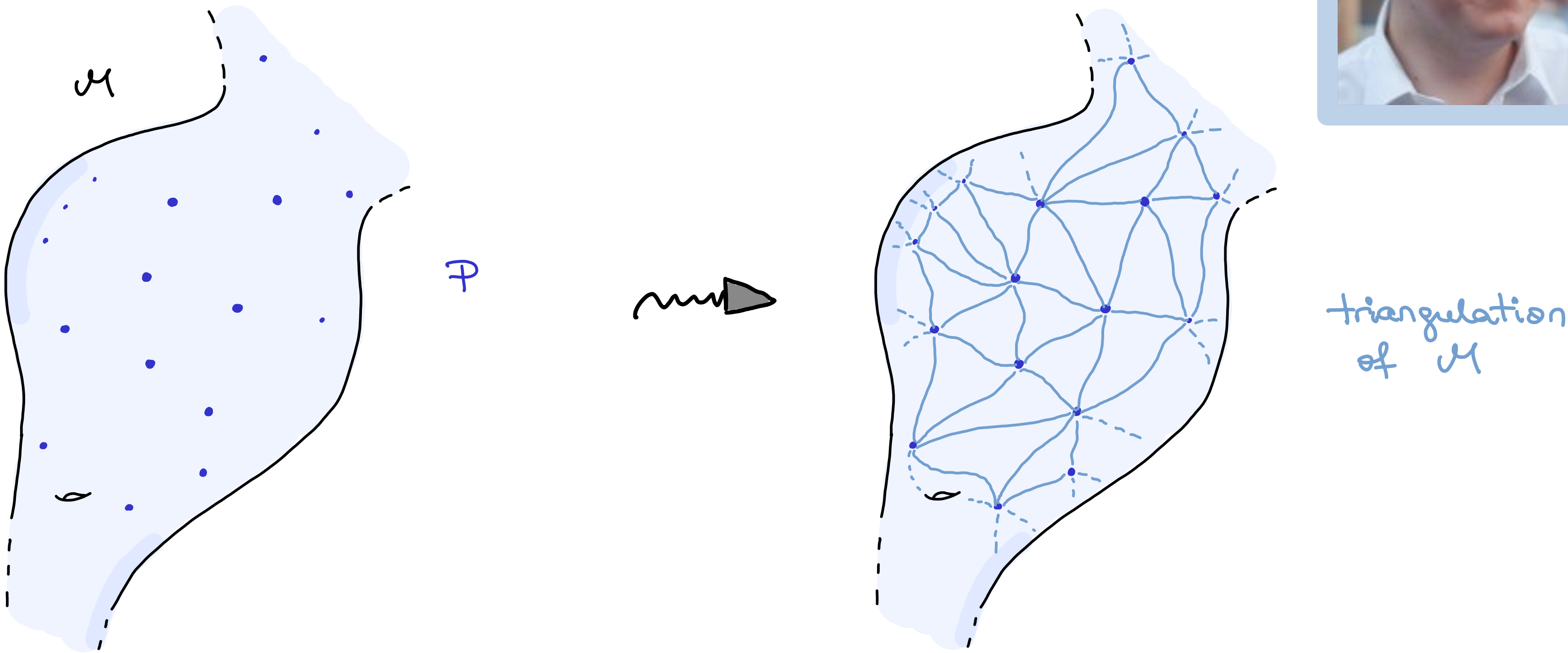


$\rightsquigarrow$ from the project

Curvature variation based adaptive sampling for Delaunay triangulations of Riemannian manifolds

by DPK, Mathijs Wintraecken





Setting

we work with SPACE OF CONSTANT NON-ZERO CURVATURE (simply connected)

Setting

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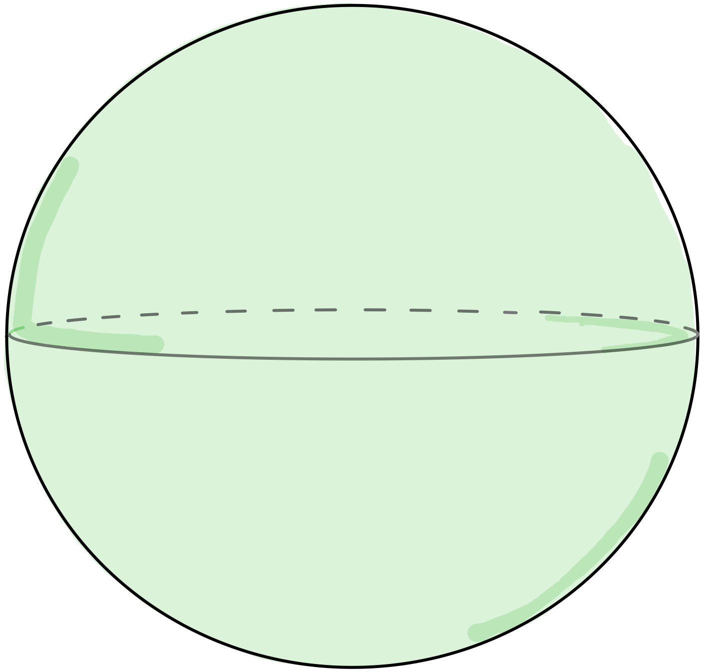
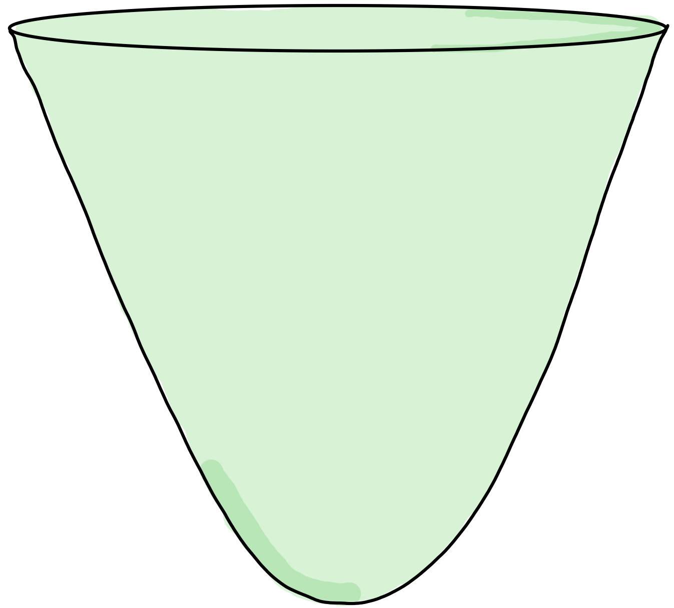


$r \cdot \mathbb{S}^n$



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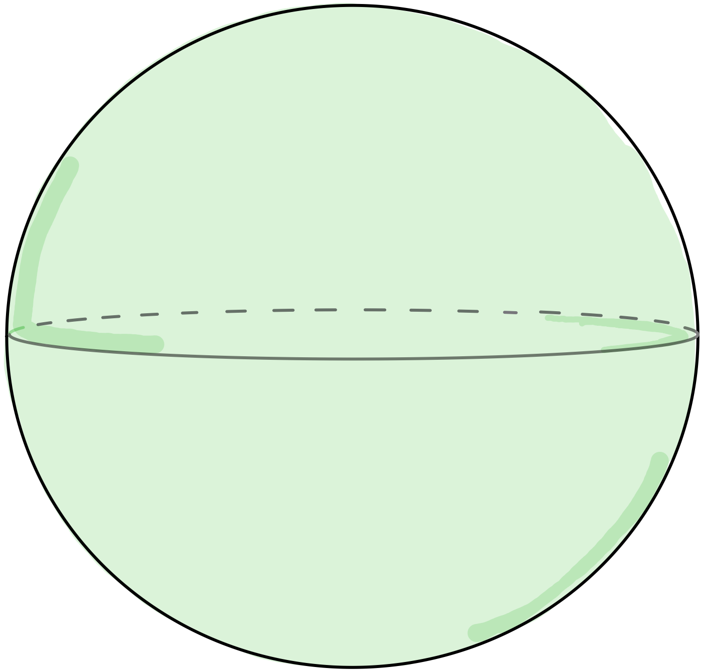
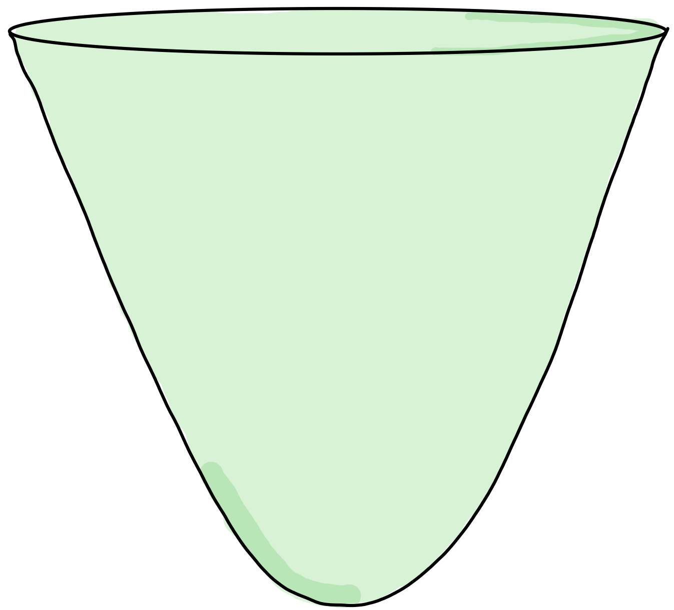


$r \cdot \mathbb{S}^n = \{ x \in \mathbb{R}^{n+1} \mid \langle x, x \rangle = r^2 \}$

$= \sum_{i=1}^{n+1} x_i^2$



$r \cdot \mathbb{H}^n$

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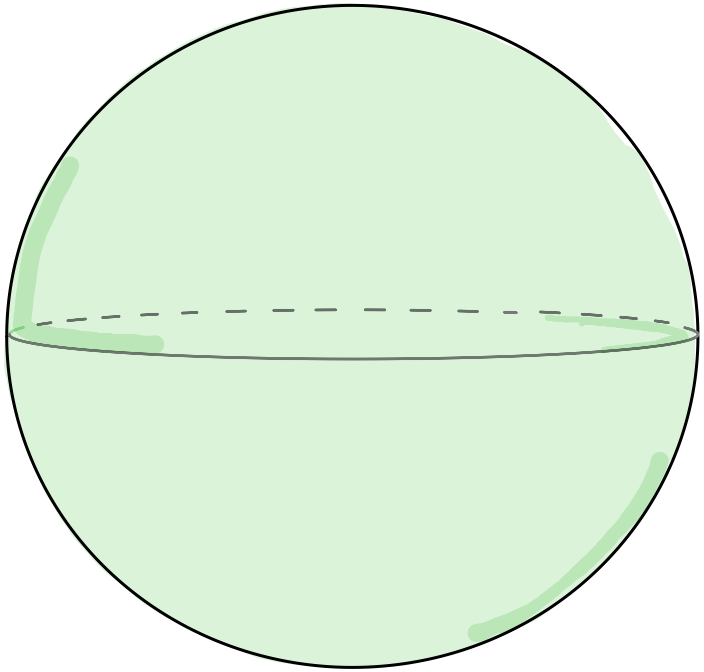
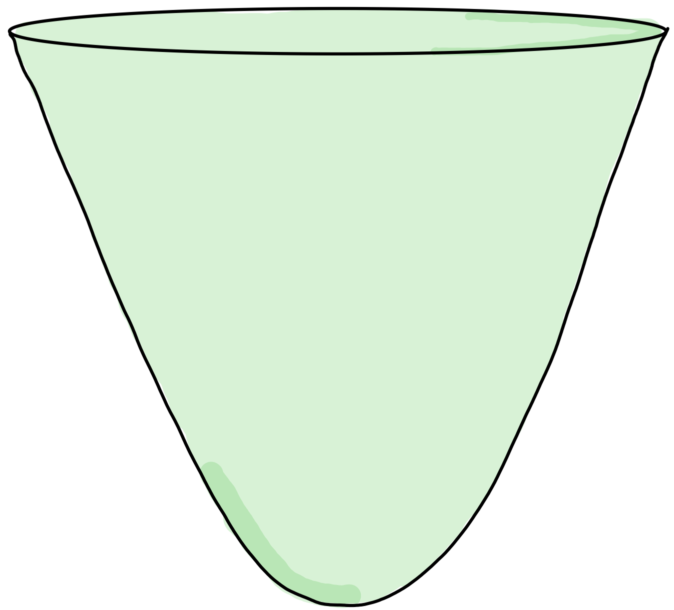


$$r \cdot \mathbb{S}^n = \{ x \in \mathbb{R}^{n+1} \mid \underbrace{\langle x, x \rangle}_{= \sum_{i=1}^{n+1} x_i^2} = r^2 \}$$



$$r \cdot \mathbb{H}^n = \{ x \in \mathbb{R}^{n,1} \mid \underbrace{\langle x, x \rangle}_{= \sum_{i=1}^{n} x_i^2 - x_{n+1}^2} = -r^2 \}$$

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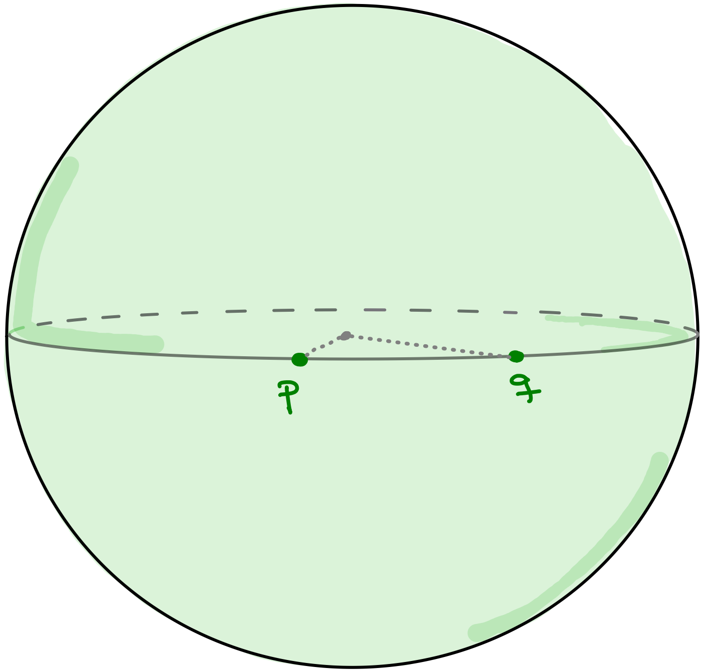
$\leadsto$ work with SPACE OF CONSTANT NON-ZERO CURVATURE (simply connected)



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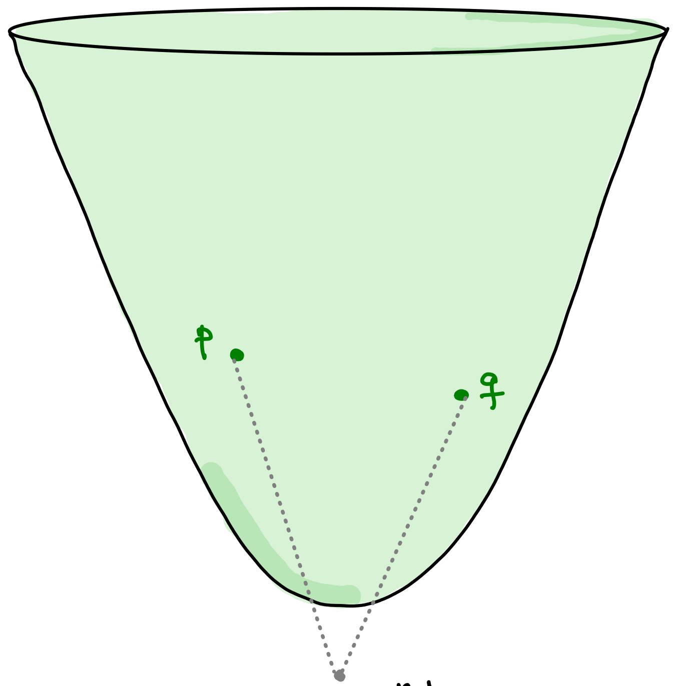

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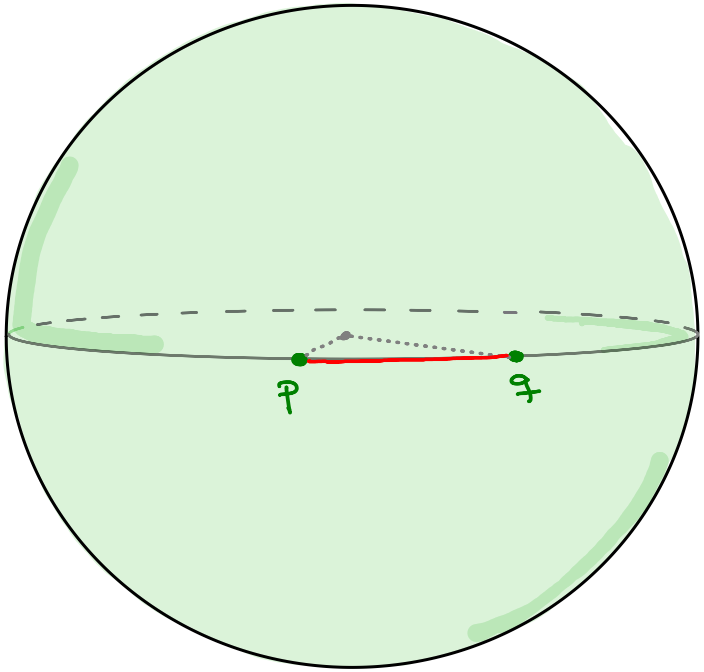

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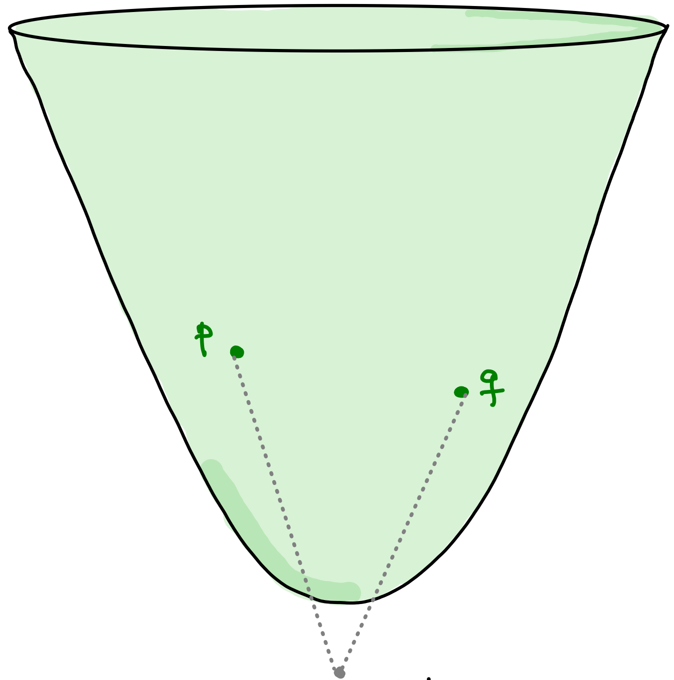
geodesics $\subseteq$ CIRCLES OF RADIUS r

$= r \cdot \mathbb{S}^n \cap \operatorname{span}\{p, q\}$



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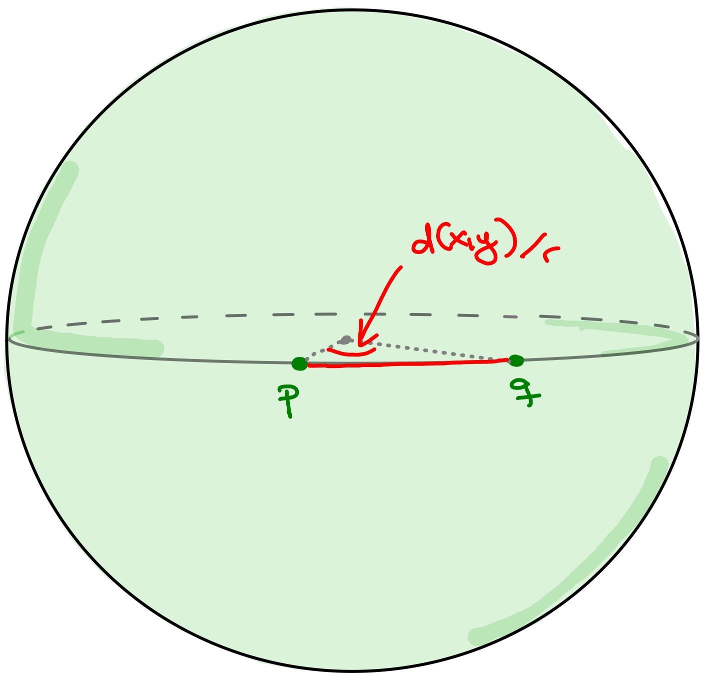

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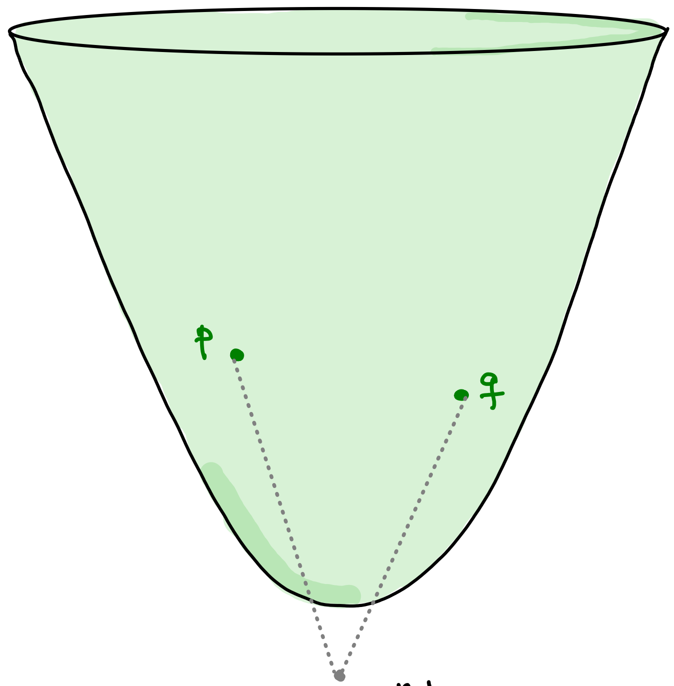

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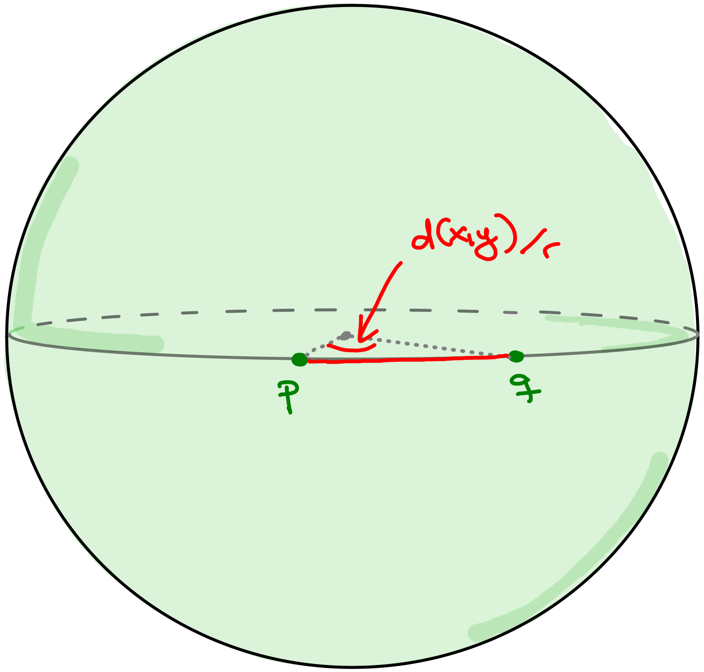

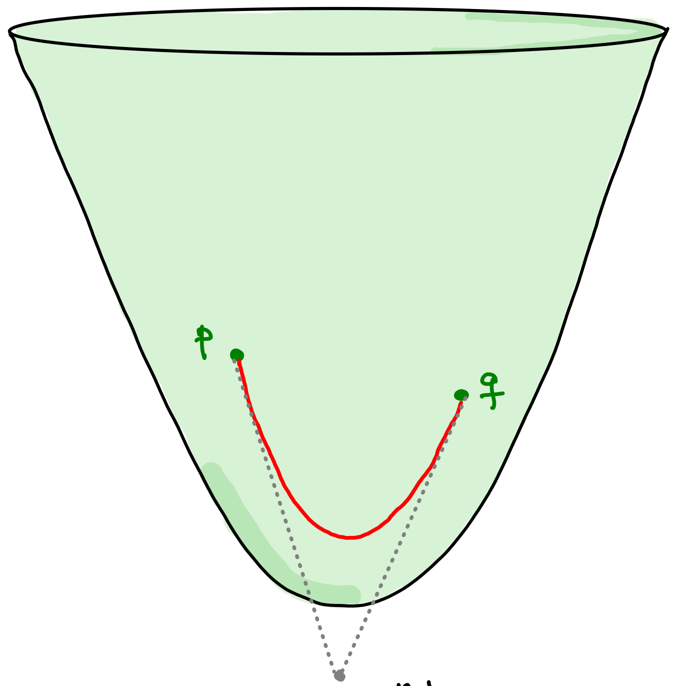

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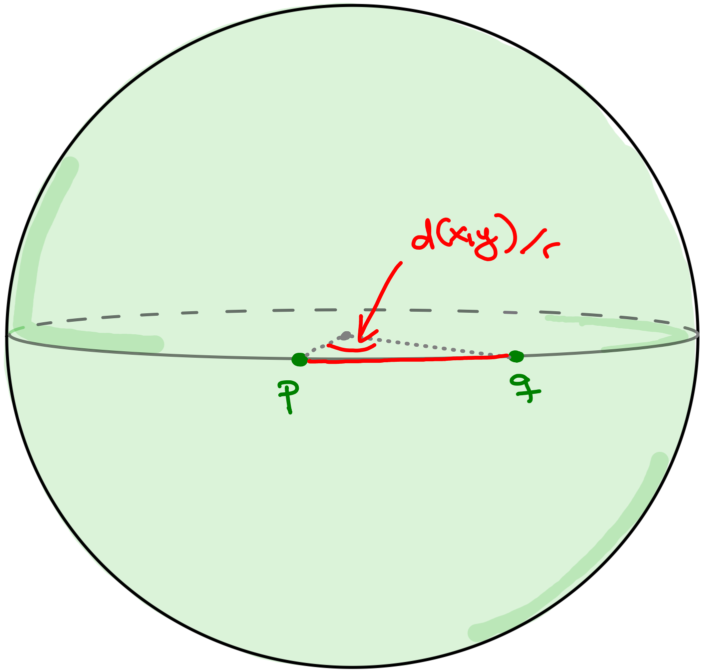

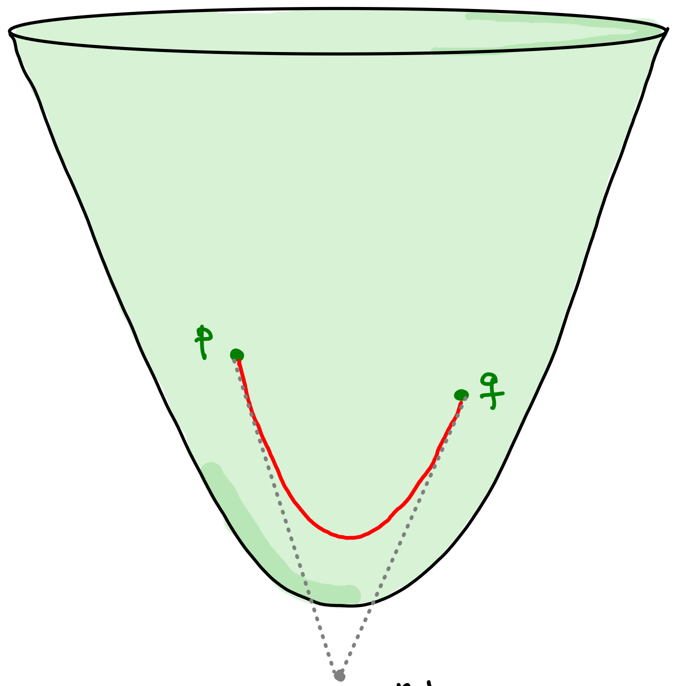

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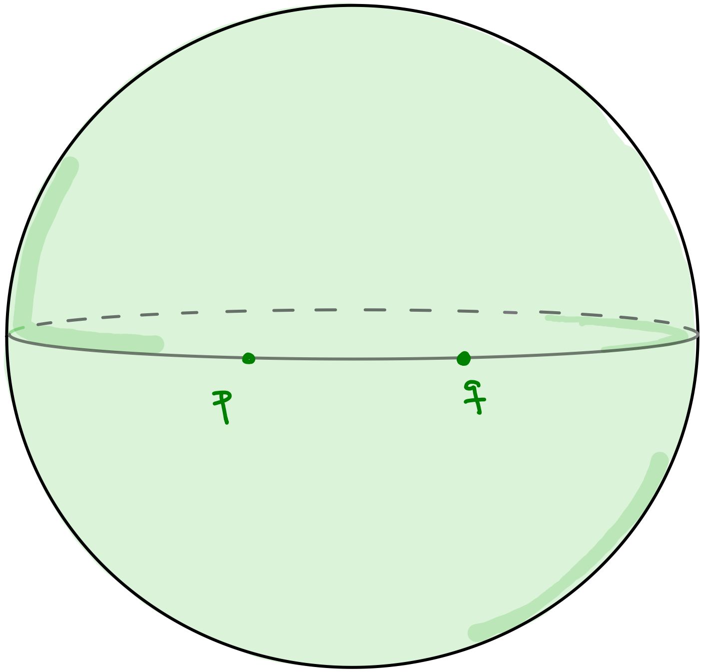
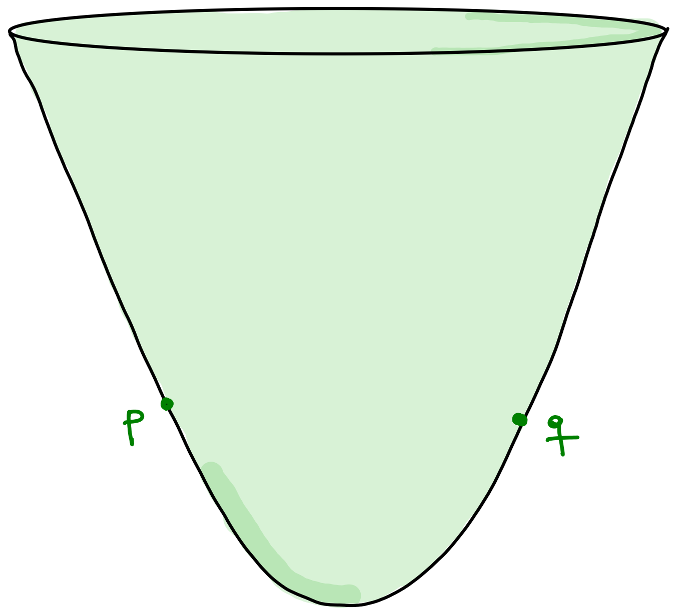
$$d(p,q) = r \cdot \operatorname{arccosh}\left(- \underbrace{\langle p/r, q/r \rangle} \right)$$

Question 1

Given $p, q \in r \cdot \mathbb{S}^n$ (resp. $r \cdot \mathbb{H}^n$),

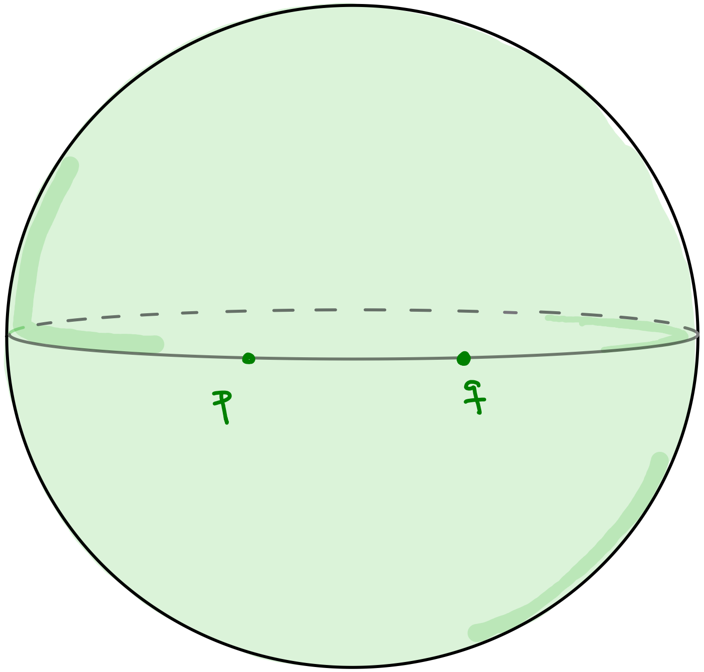


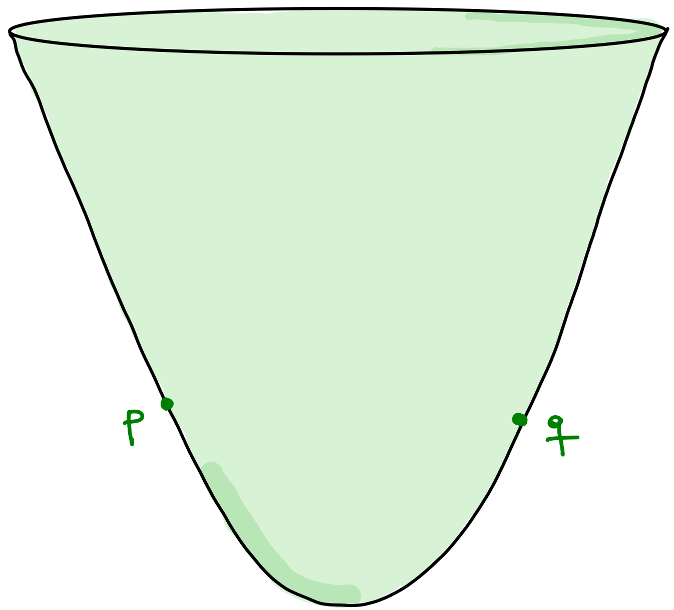


Question 1

Given $p, q \in r \cdot \mathbb{S}^n$ (resp. $r \cdot \mathbb{H}^n$), what is

$$\{ x \in r \cdot \mathbb{S}^n \text{ (resp. } r \cdot \mathbb{H}^n) \mid d(x,p) = d(x,q) \} \ ?$$



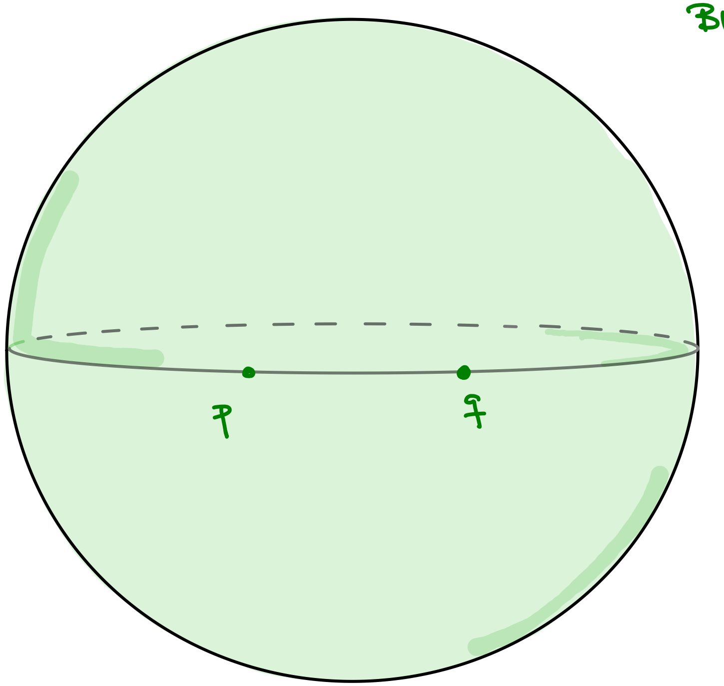


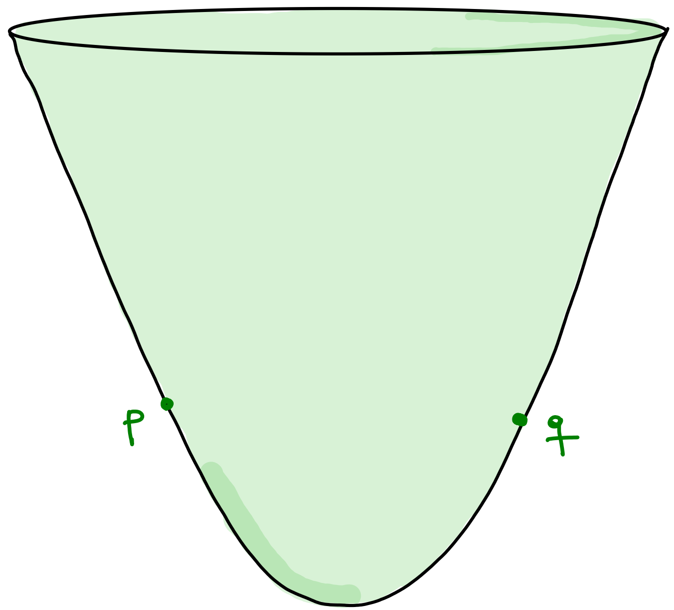
Question 1

Given $p,q \in r\cdot\mathbb{S}^n$ (resp. $r\cdot\mathbb{H}^n$), what is

$$\mathcal{B}(p,q) := \{x \in r\cdot\mathbb{S}^n \text{ (resp. } r\cdot\mathbb{H}^n) \mid d(x,p) = d(x,q)\} \,?$$

BISECTOR of p,q





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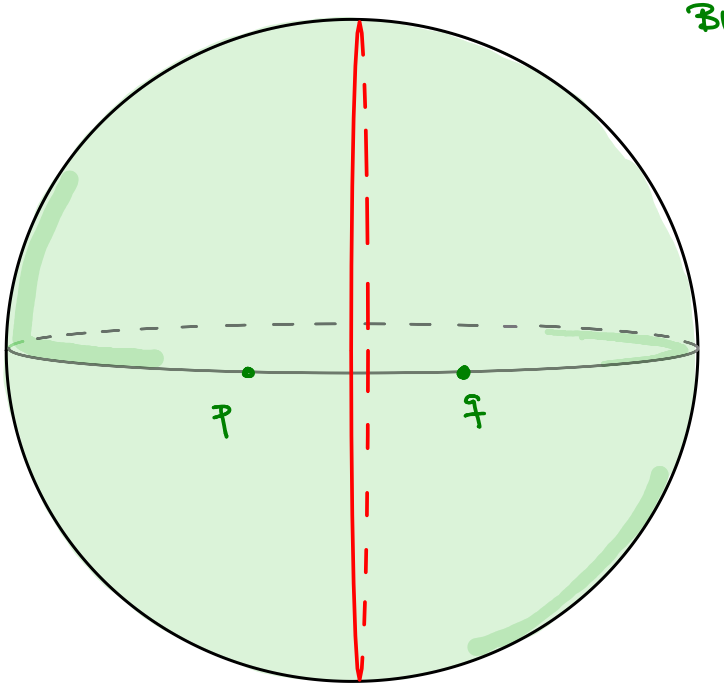
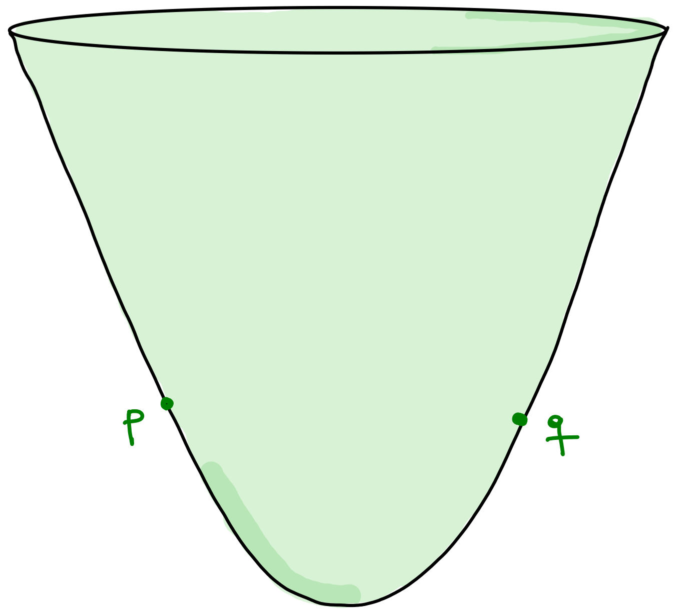


circle (resp. $\approx r\cdot\mathbb{S}^{n-1}$)



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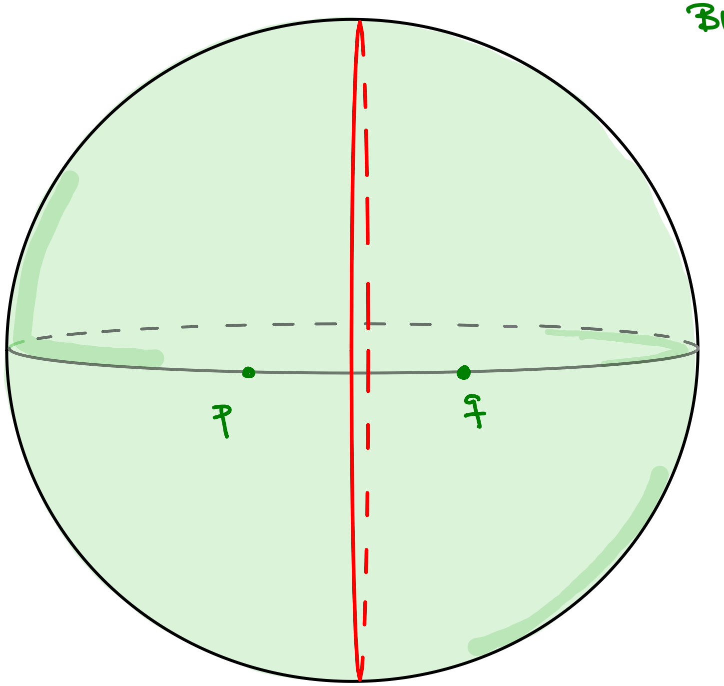

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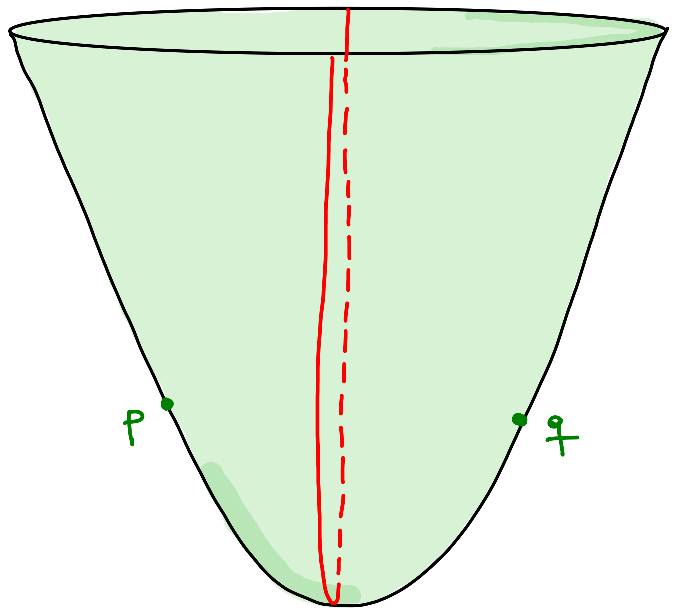

hyperbola (resp. $\approx r\cdot\mathbb{H}^{n-1}$)

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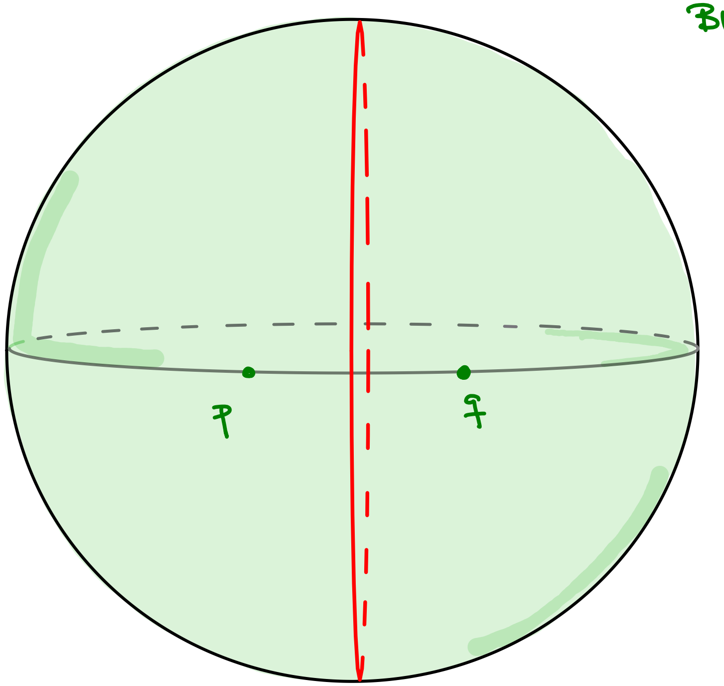
BISECTOR of p, q



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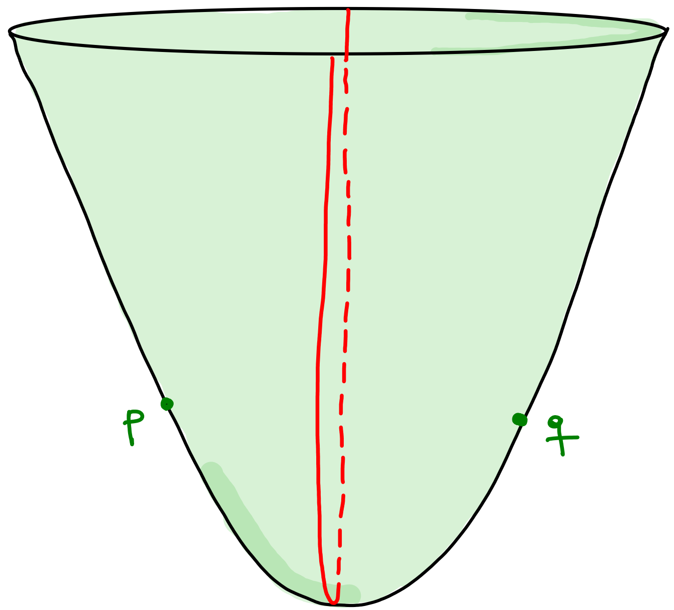

hyperbola (resp. $\approx r \cdot \mathbb{H}^{n-1}$)

$$\mathcal{B}(p,q) = r \cdot \mathbb{S}^n \text{ (resp. } r \cdot \mathbb{H}^n) \cap \operatorname{span}\{p-q\}^\perp$$

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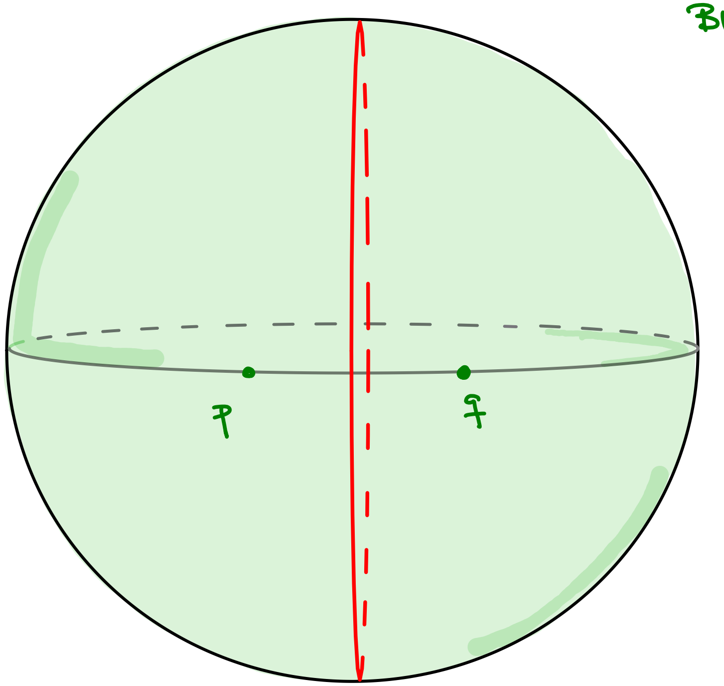
BISECTOR of p,q



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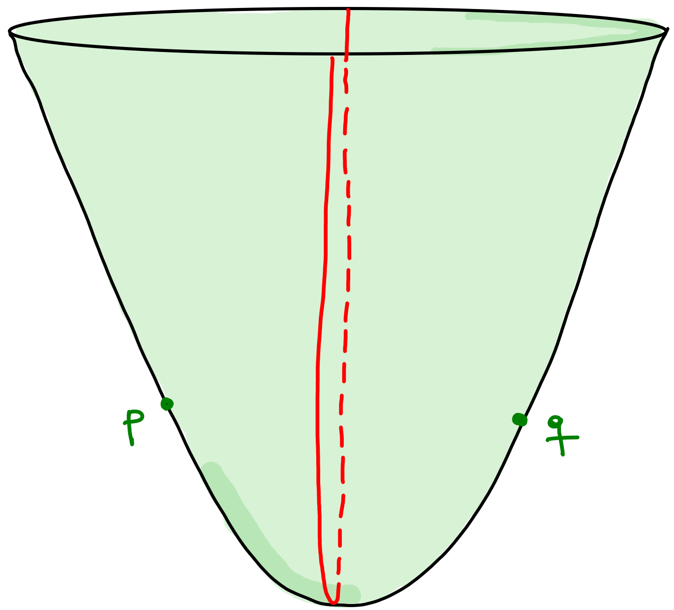

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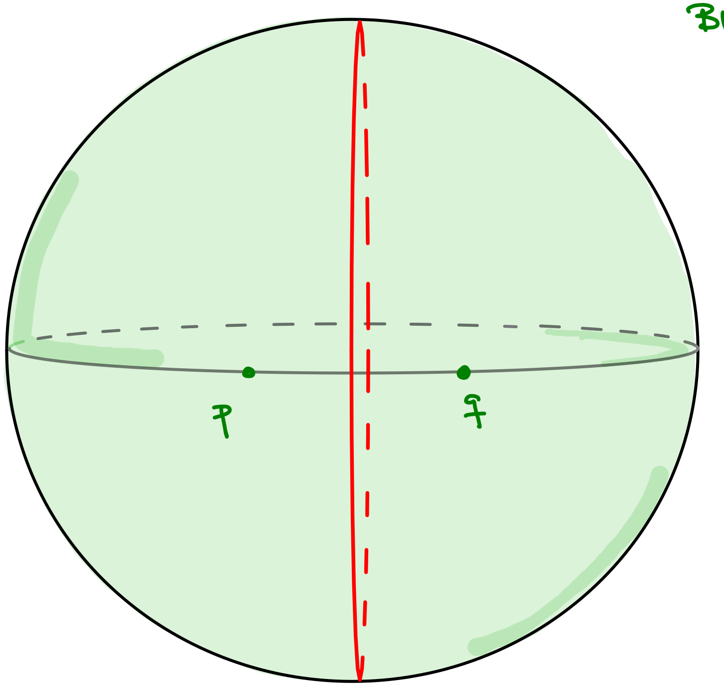
$$\mathcal{B}(p,q) := \{\, x \in r\cdot\mathbb{S}^n \ (\text{resp. } r\cdot\mathbb{H}^n) \mid |d(x,p) - d(x,q)| = 0 \,\} \ ?$$

BISECTOR of p,q



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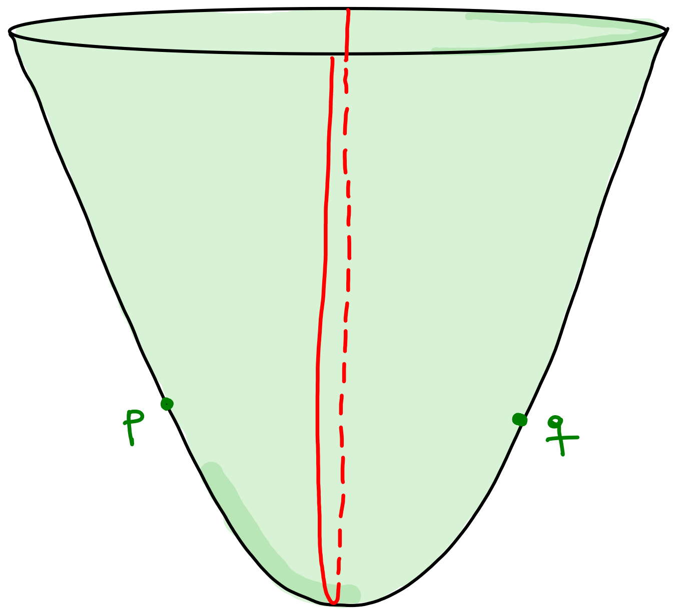

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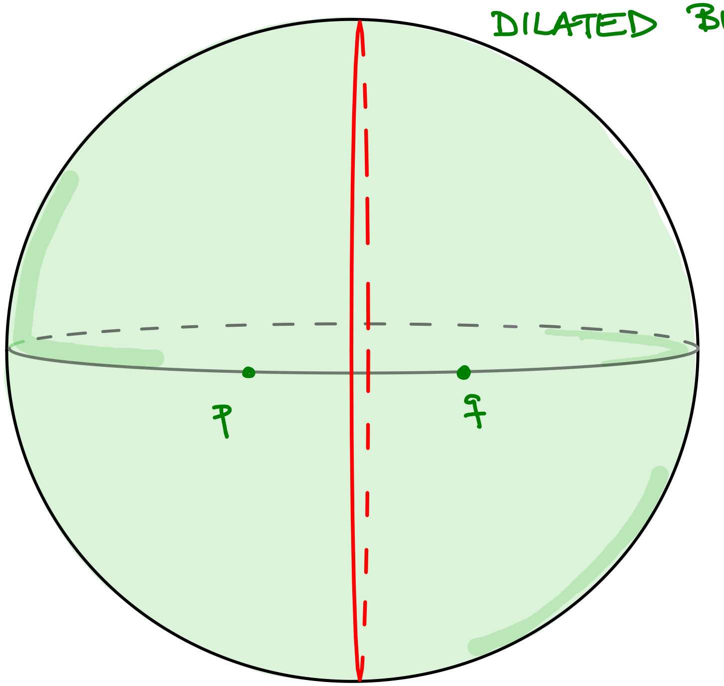
Question ~~1~~ 2

Given $p,q \in r\cdot\mathbb{S}^n$ (resp. $r\cdot\mathbb{H}^n$), what is

very small

$$\mathcal{B}_\nu(p,q) := \{x \in r\cdot\mathbb{S}^n \text{ (resp. } r\cdot\mathbb{H}^n) \mid |d(x,p) - d(x,q)| \cancel{=0} \leq 2\nu\} \;?$$

DILATED BISECTOR of p,q



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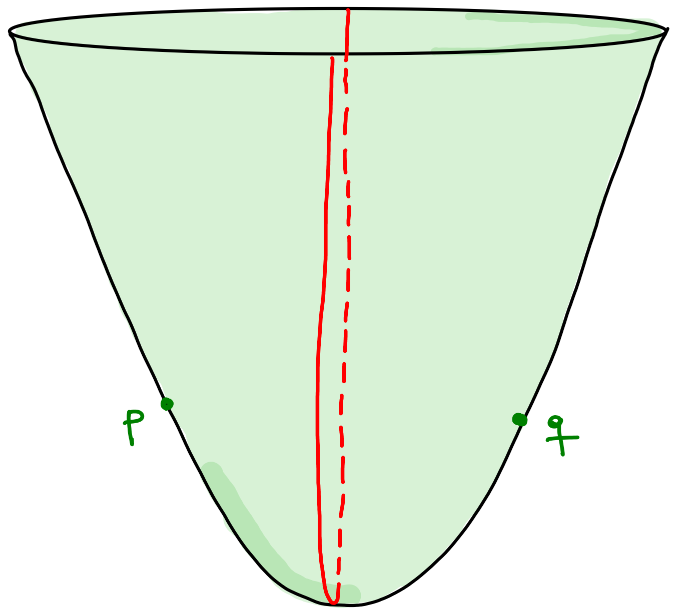

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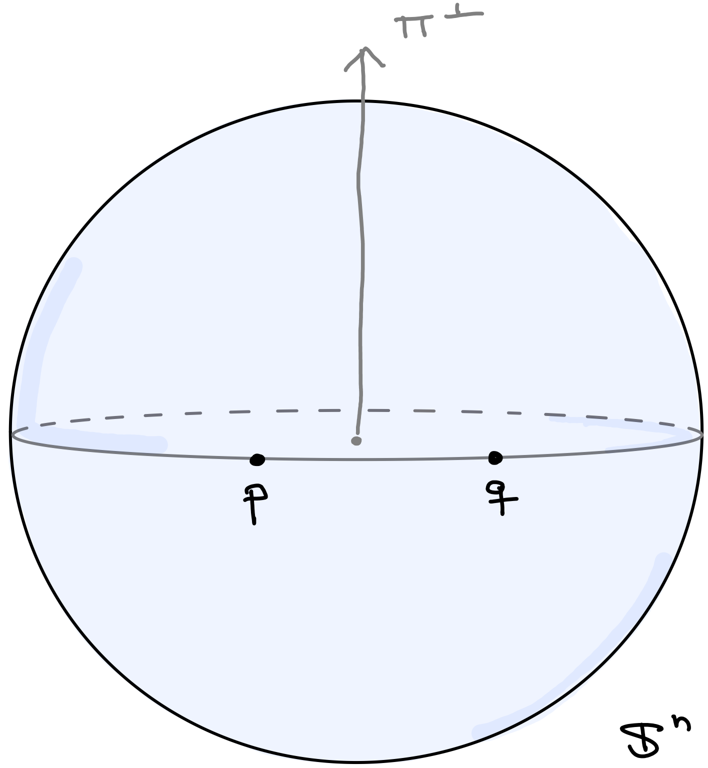
The mysterious $\mathbb{S}_v(p,q)$

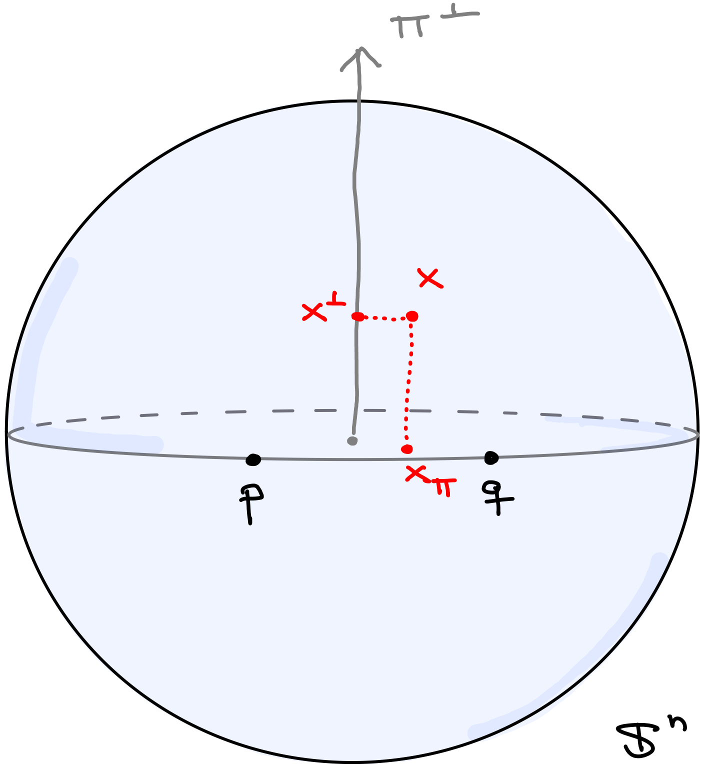
1) set $r = 1 \implies d(a,b) = \arccos\langle a,b\rangle$



The mysterious $\mathbb{S}_\nu(p,q)$

1) set $r = 1 \implies d(a,b) = \arccos\langle a,b\rangle$

2) $\Pi := \operatorname{span}\{p,q\} \implies x = x_\pi + x^\perp$, $x_\pi \in \Pi$, $x^\perp \perp \Pi$



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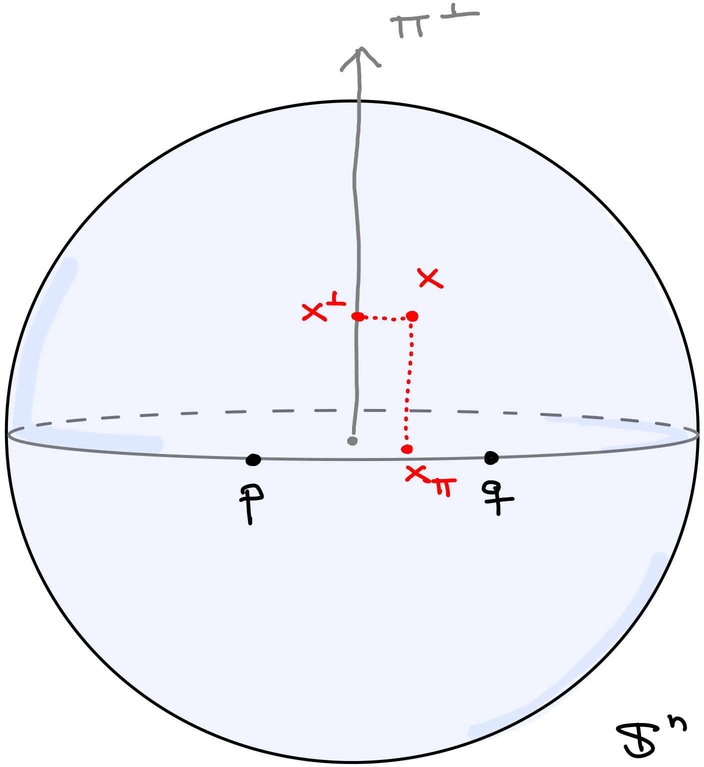
$$\Rightarrow |d(x,p) - d(x,q)| = |\arccos\langle x_\pi, p\rangle - \arccos\langle x_\pi, q\rangle|$$



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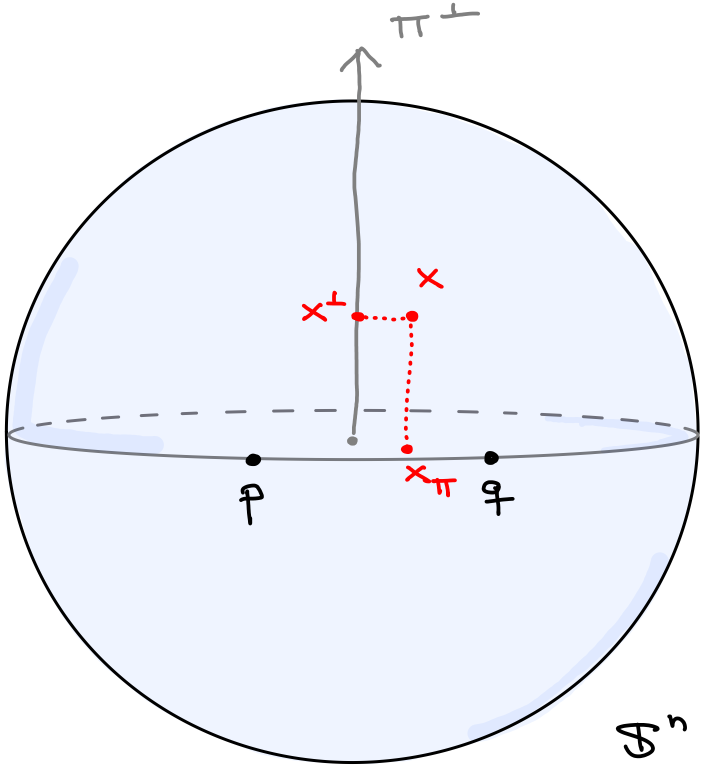
$$\Rightarrow \quad |d(x,p) - d(x,q)| = \underbrace{|\arccos\langle x_\pi,p\rangle - \arccos\langle x_\pi,q\rangle|}$$

a planar problem!



The mysterious $\mathscr{L}_\nu(p,q)$

1) set $r = 1 \implies d(a,b) = \arccos\langle a,b\rangle$

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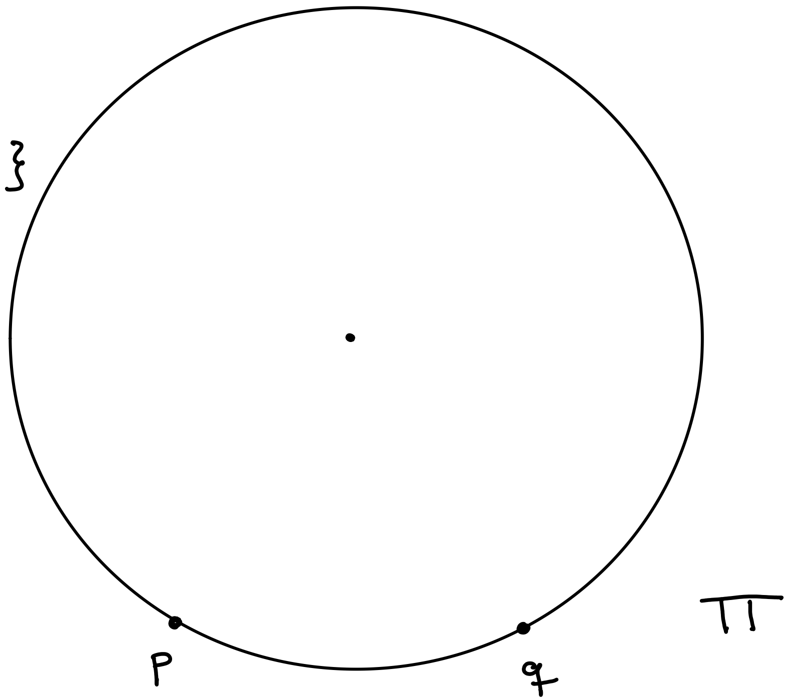
$$\implies |d(x,p) - d(x,q)| = \underbrace{|\arccos\langle x_\pi, p\rangle - \arccos\langle x_\pi, q\rangle|}$$

a planar problem!

3) $\underset{\circ\,\circ\,\circ}{???} = \{x \in \Pi \mid |\arccos\langle x,p\rangle - \arccos\langle x,q\rangle| \le 2\nu\}$



The mysterious $\mathcal{E}_\nu(p,q)$

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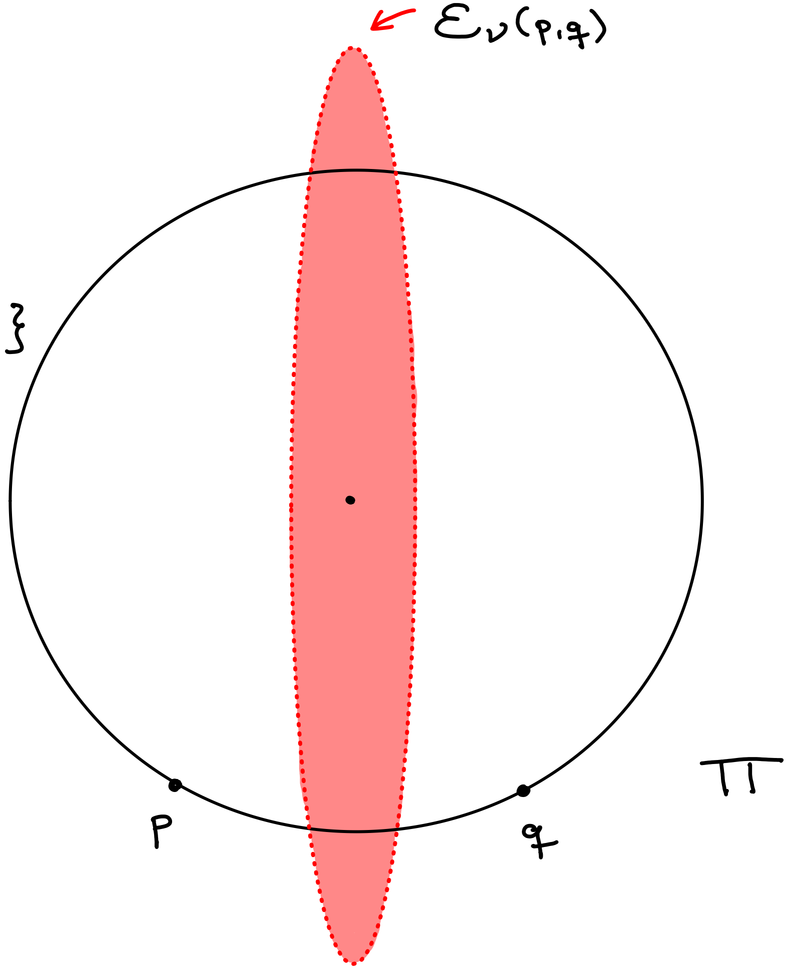
a planar problem!

3) $\mathcal{E}_\nu(p,q) := \{x \in \Pi \mid |\arccos\langle x,p\rangle - \arccos\langle x,q\rangle| \leq 2\nu\}$



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a planar problem!

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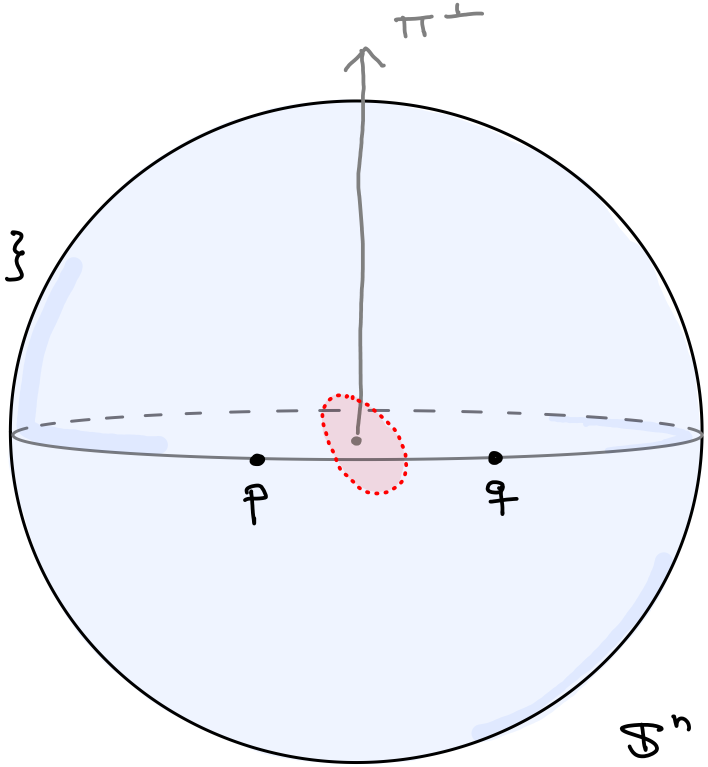

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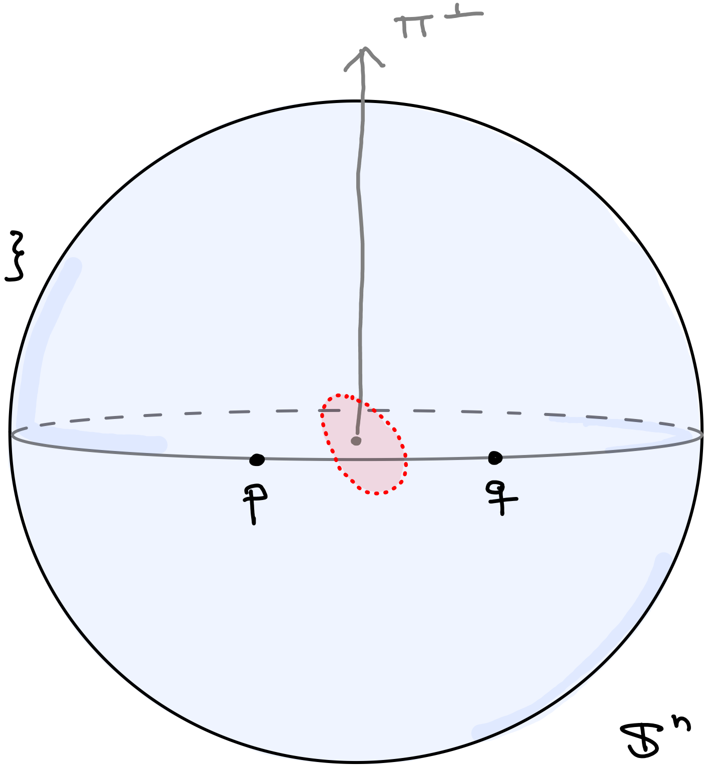

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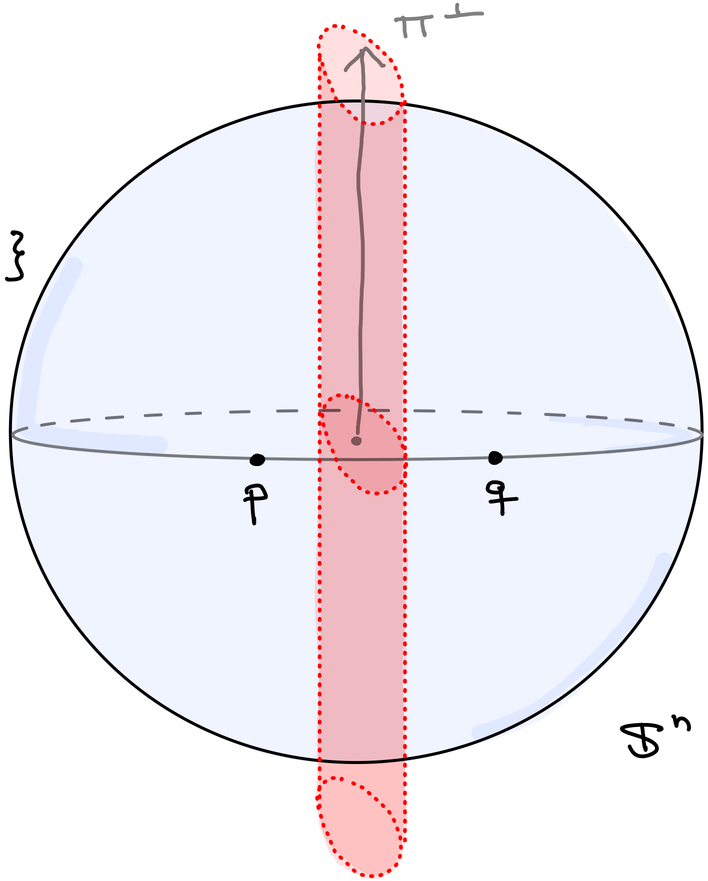

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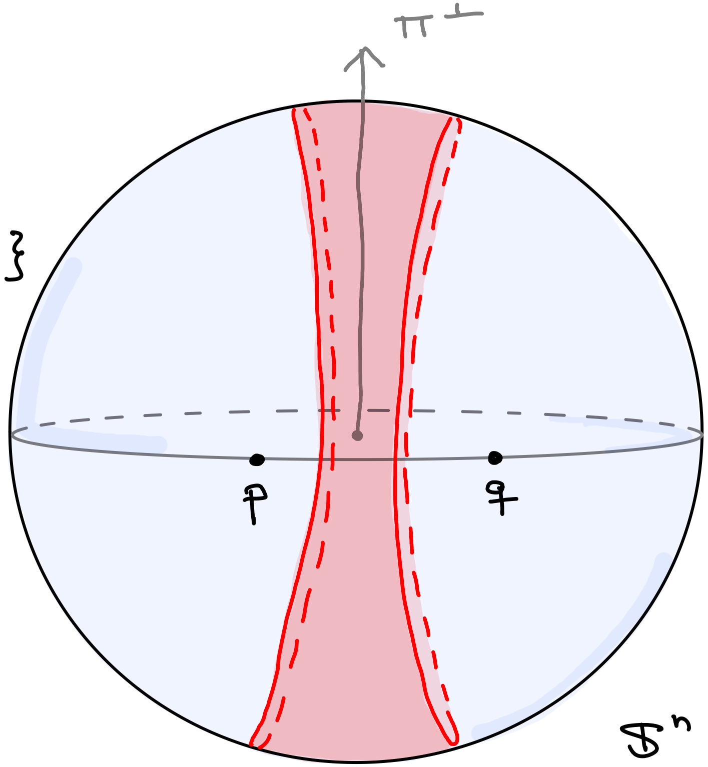

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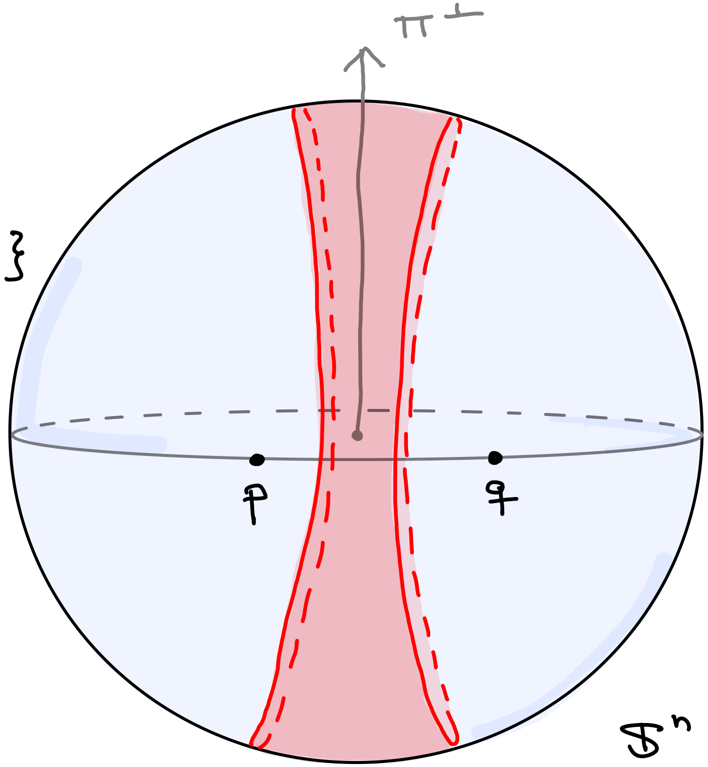

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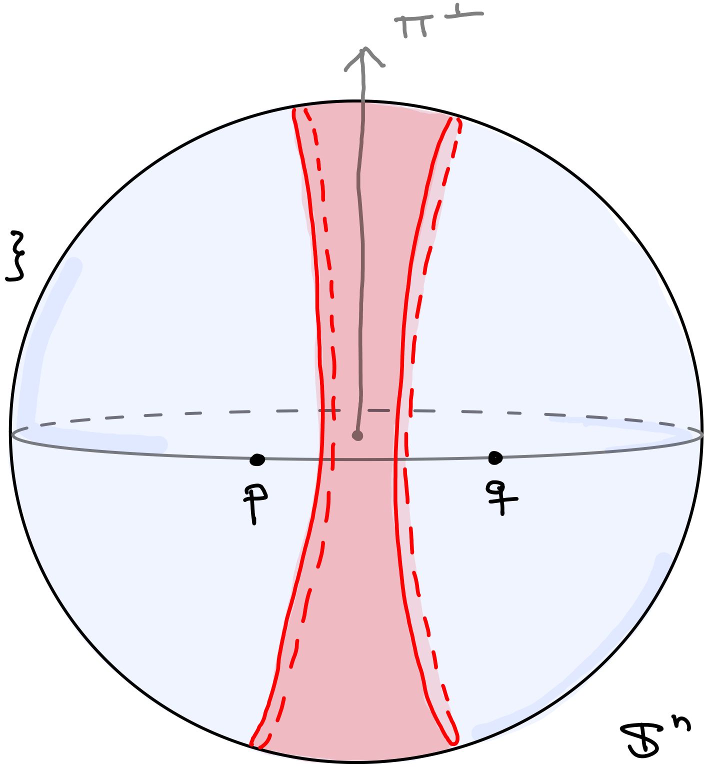

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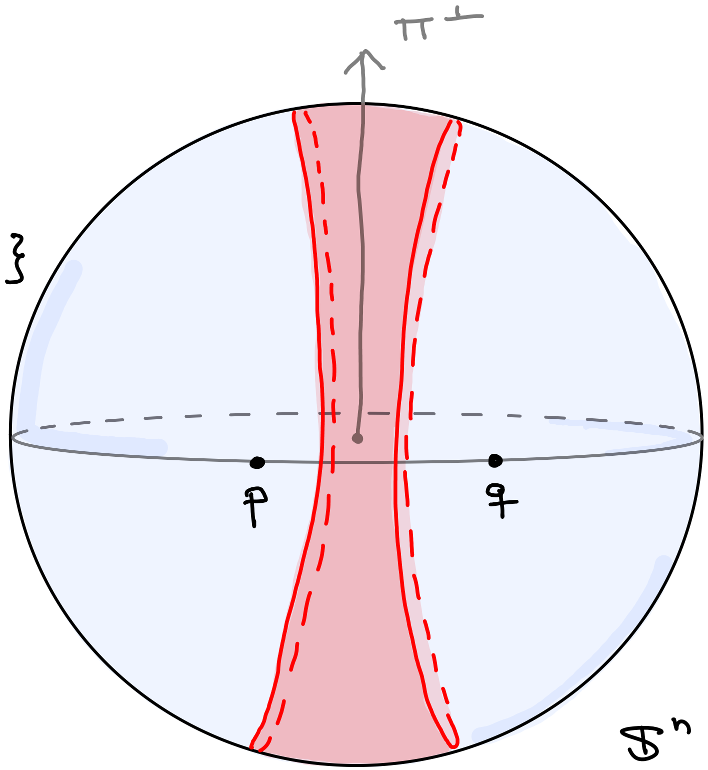

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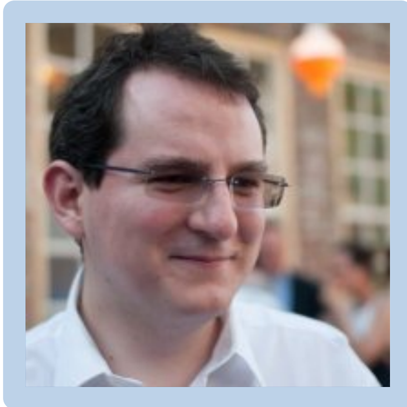
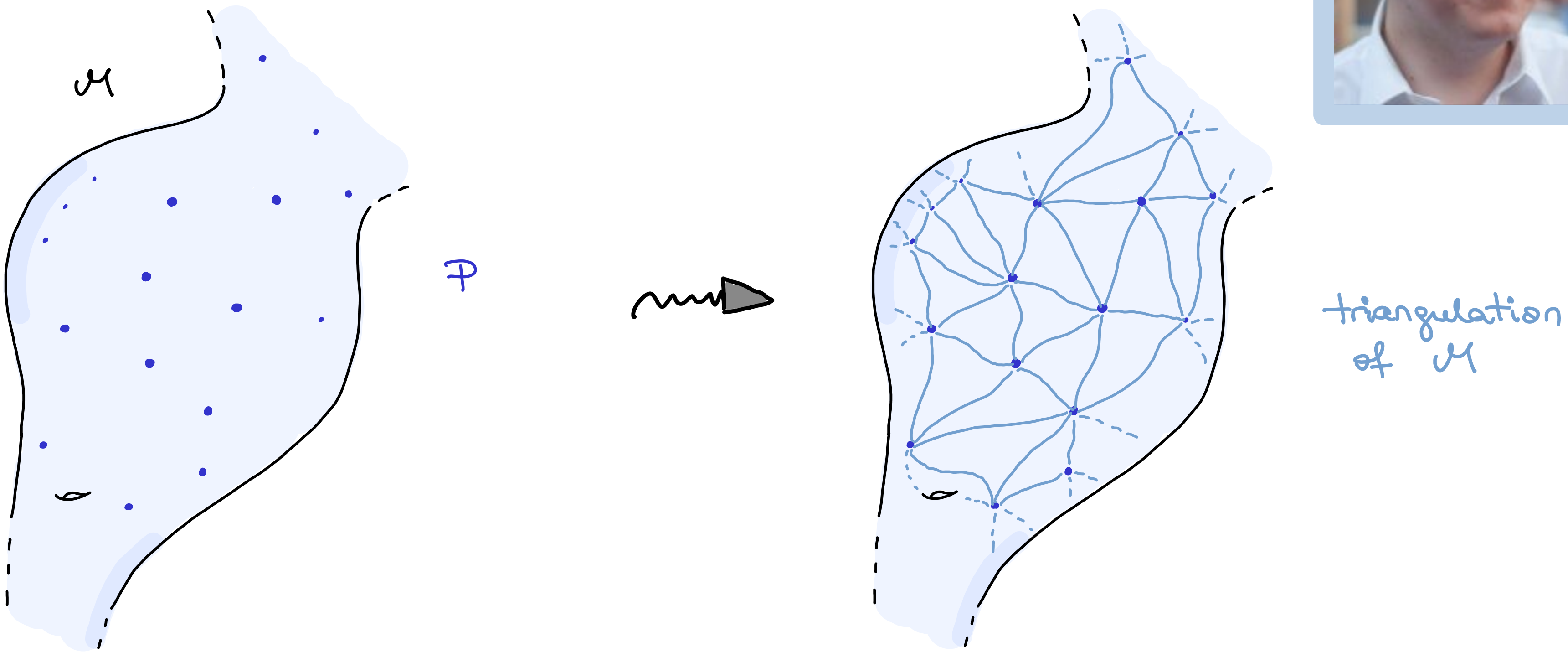
* In $\mathbb{R}^n$: can't reduce to planar problem, solution using cylindrical coordinates

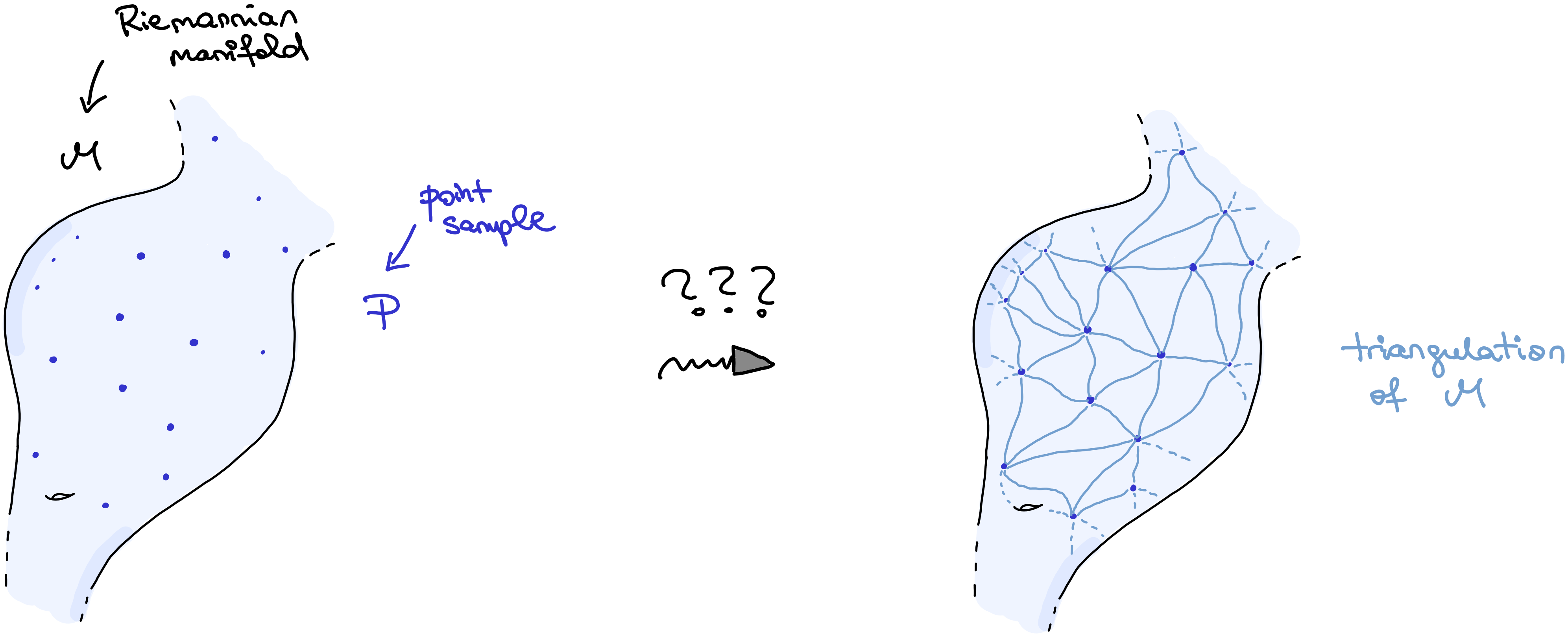
The project

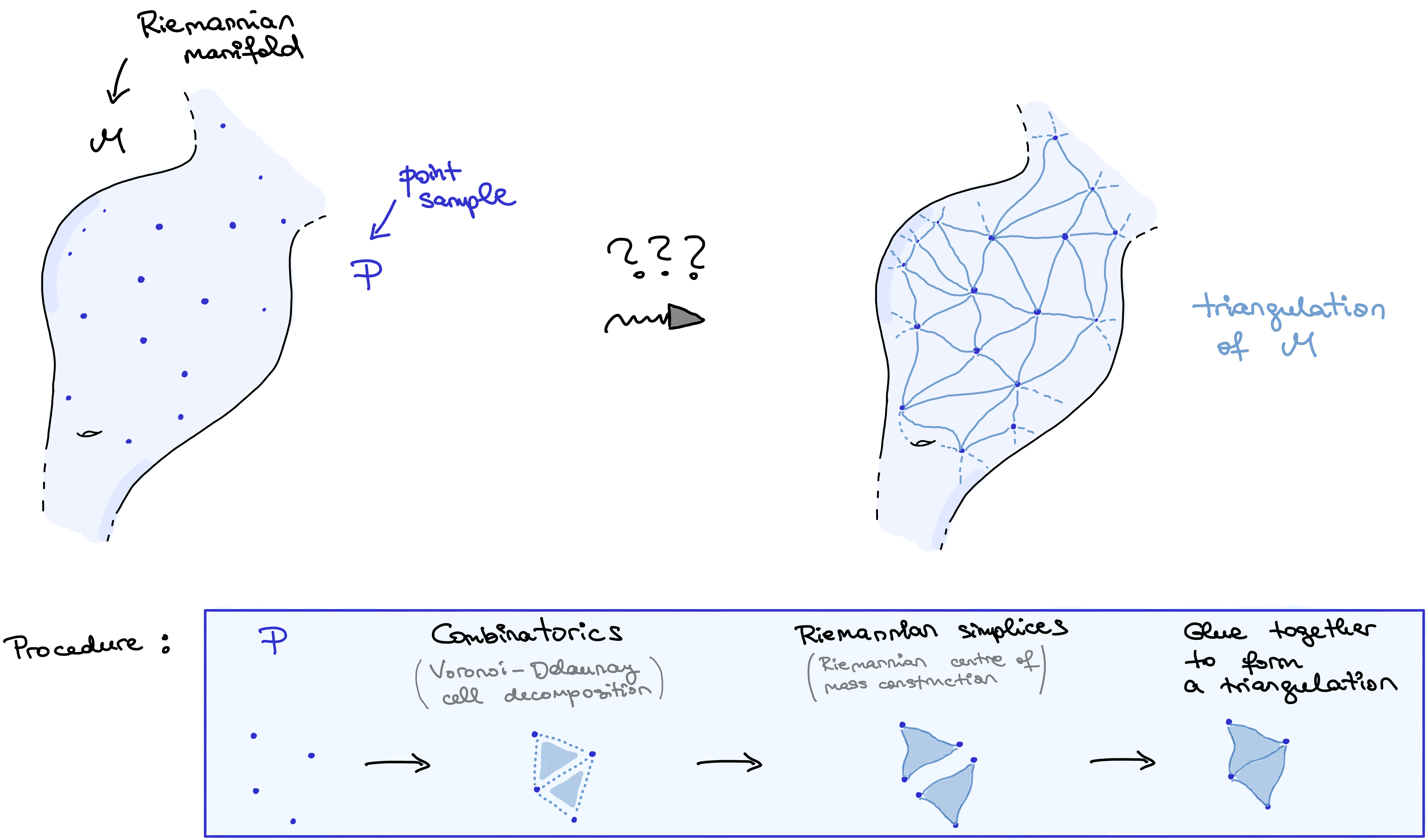
Curvature variation based adaptive sampling for Delaunay triangulations of Riemannian manifolds

by DPK, Mathijs Wintraecken






Riemannian manifold
$\mathcal{M}$
point sample
$\mathcal{P}$
???
triangulation of $\mathcal{M}$



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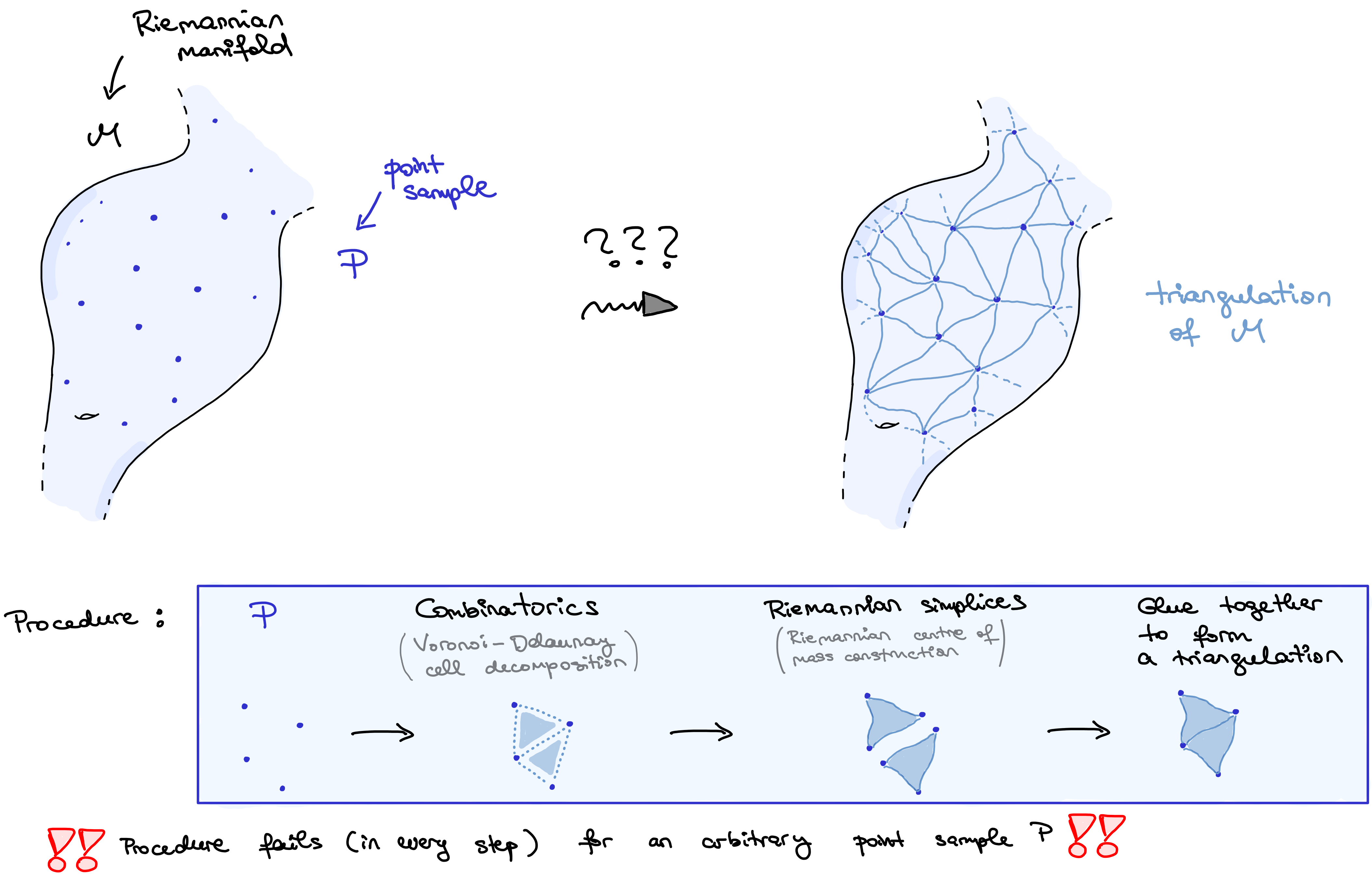
Procedure :
$\mathcal{P}$
Combinatorics
(Voronoi-Delaunay cell decomposition)
Riemannian simplices
(Riemannian centre of mass construction)
Glue together to form a triangulation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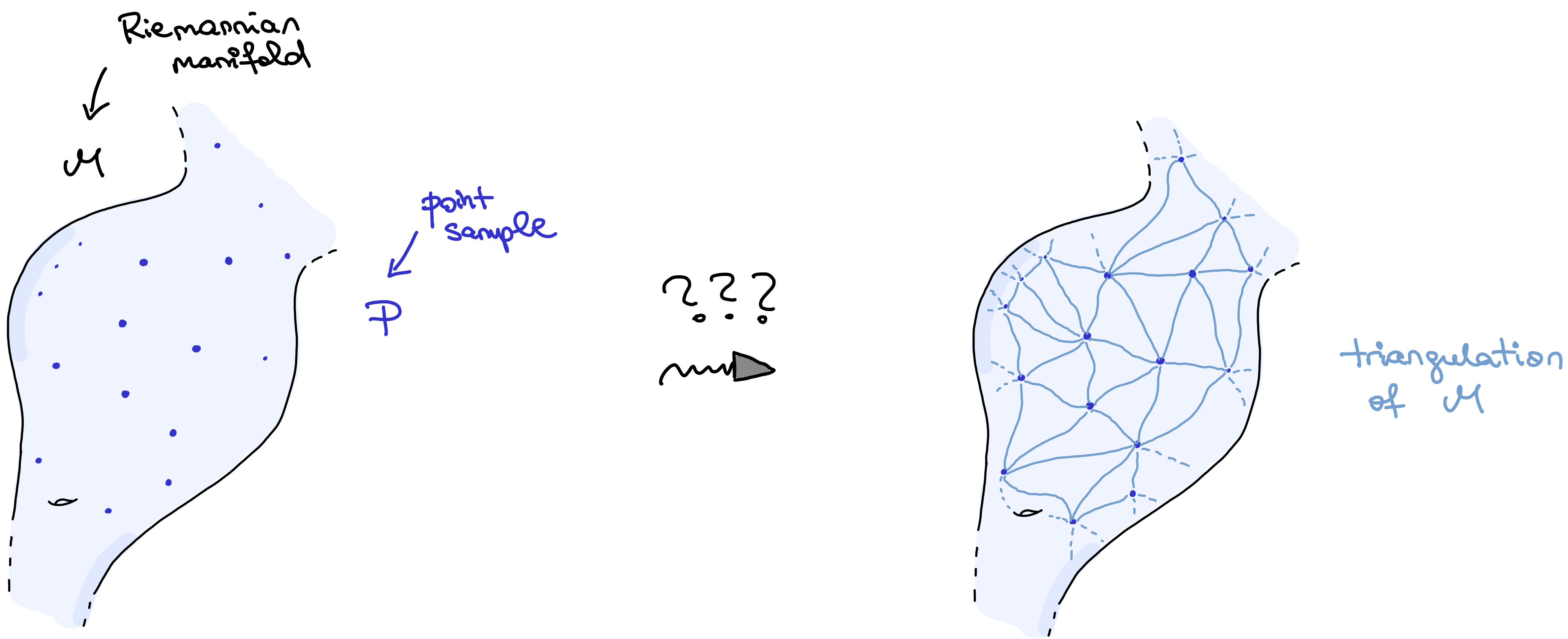
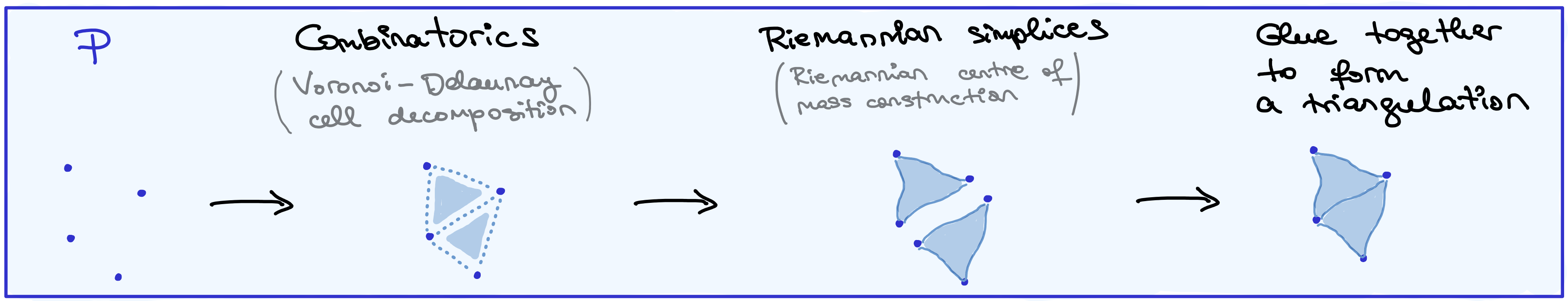
Procedure fails (in every step) for an arbitrary point sample $\mathcal{P}$



Procedure :

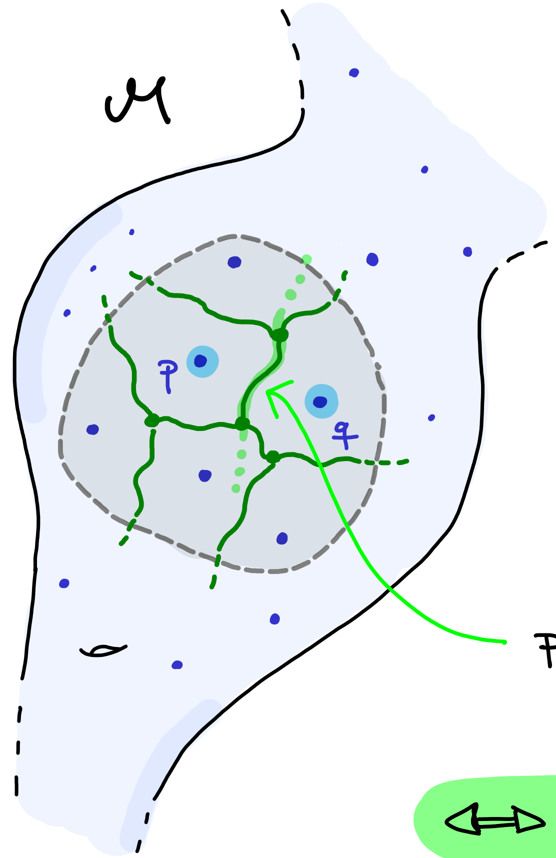


!! Procedure fails (in every step) for an arbitrary point sample $\mathcal{P}$!!

Our contribution: We provide quality criteria on $\mathcal{P}$ that guarantee that the procedure succeeds. These criteria are easy to satisfy if $\mathcal{M}$ has locally almost constant curvature.

(1) work with the Voronoi diagram

- (lower-dimensional) faces lie in intersections of bisectors



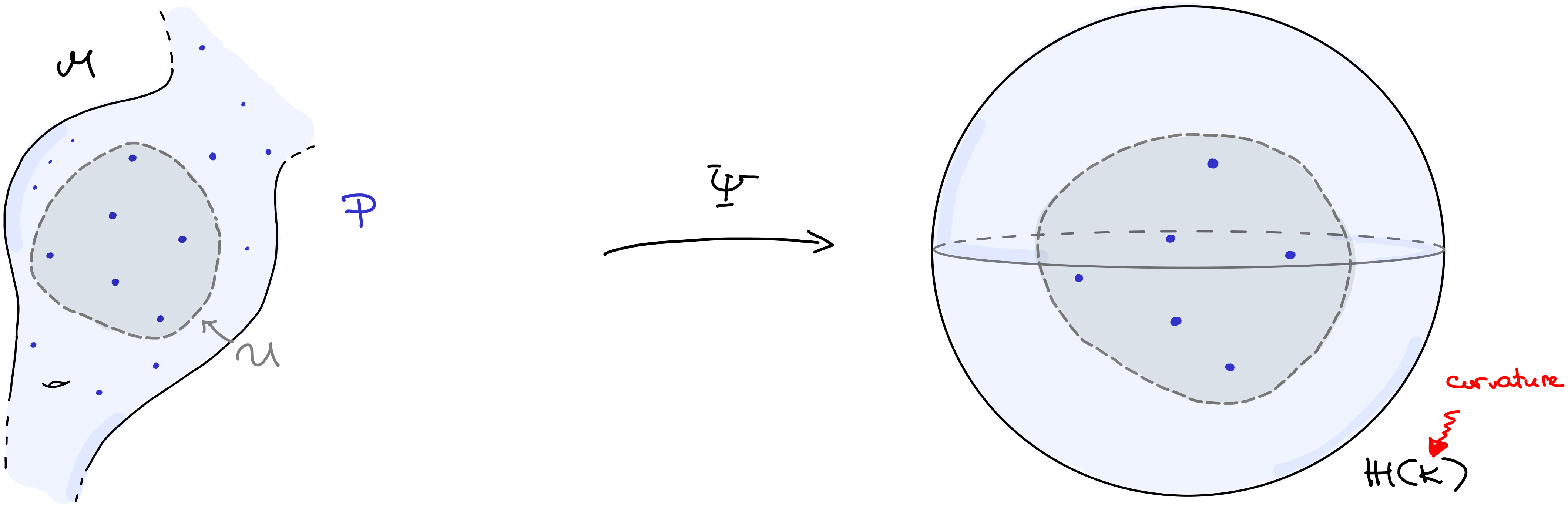
$\mathcal{P}$

points satisfy

$$d_{\mathcal{M}}(x,p) = d_{\mathcal{M}}(x,q)$$

$$\Longleftrightarrow \; |d_{\mathcal{M}}(x,p) - d_{\mathcal{M}}(x,q)| = 0$$

② Compare $\mathcal{M}$ $(\mathcal{U})$ to a space of constant non-zero curvature



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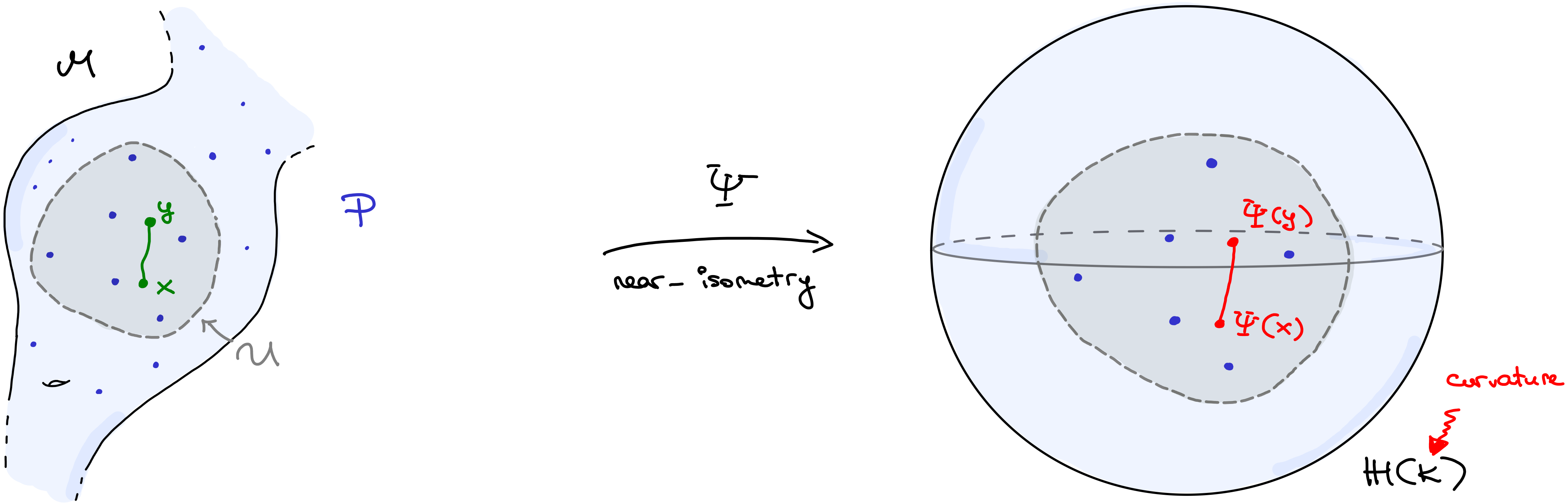

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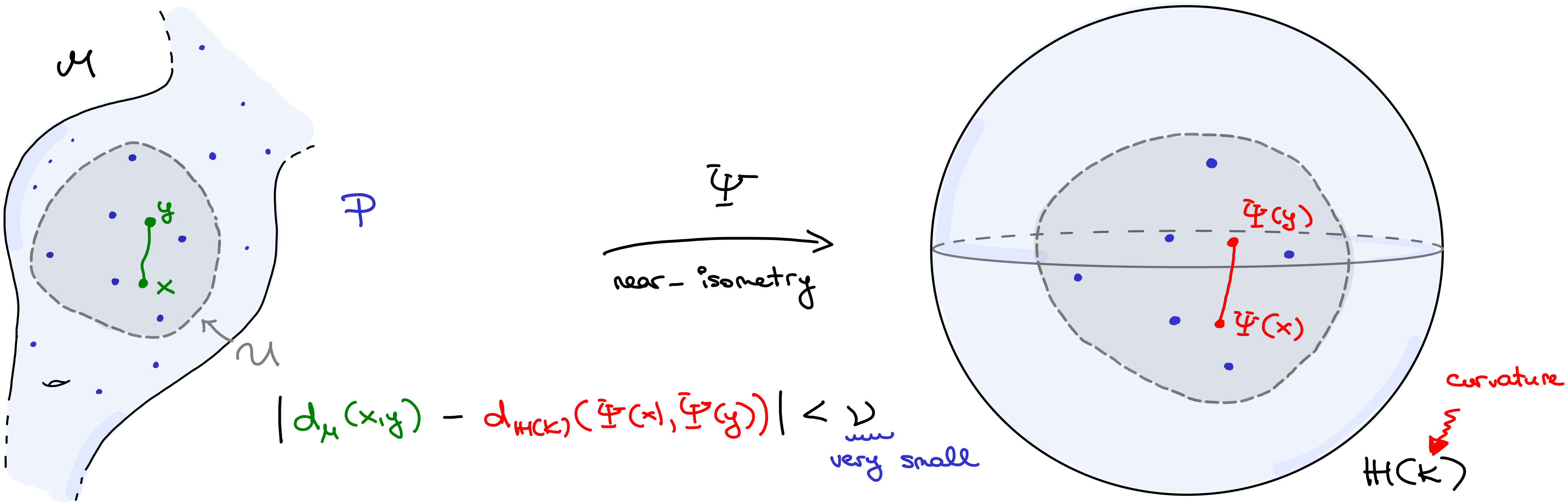
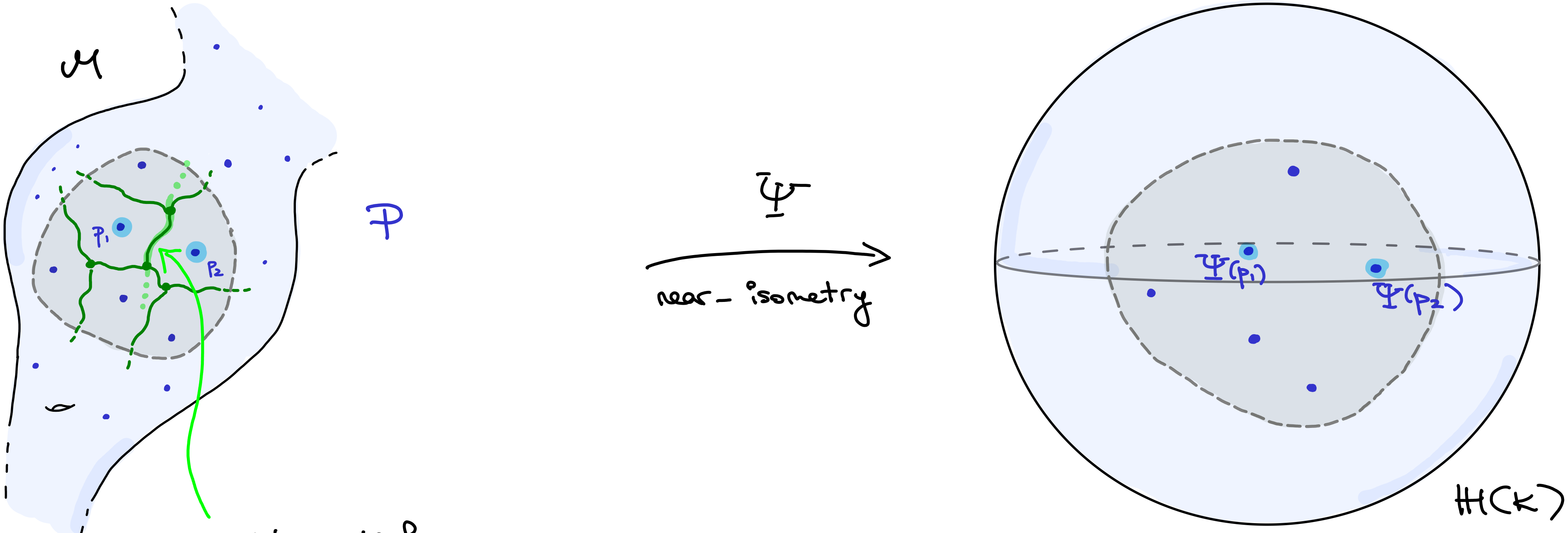


$$\left| d_{\mathcal{M}}(x,y) - d_{\mathbb{H}(K)}(\Psi(x),\Psi(y)) \right| < \underbrace{\nu}_{\text{very small}}$$



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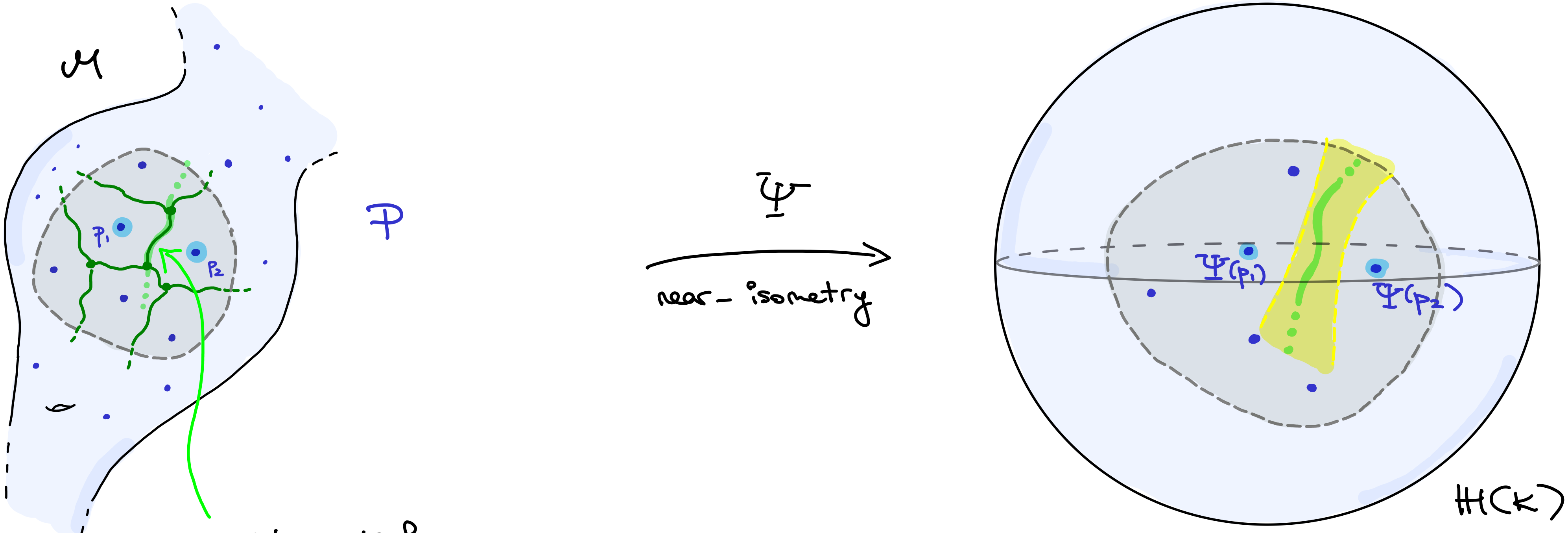

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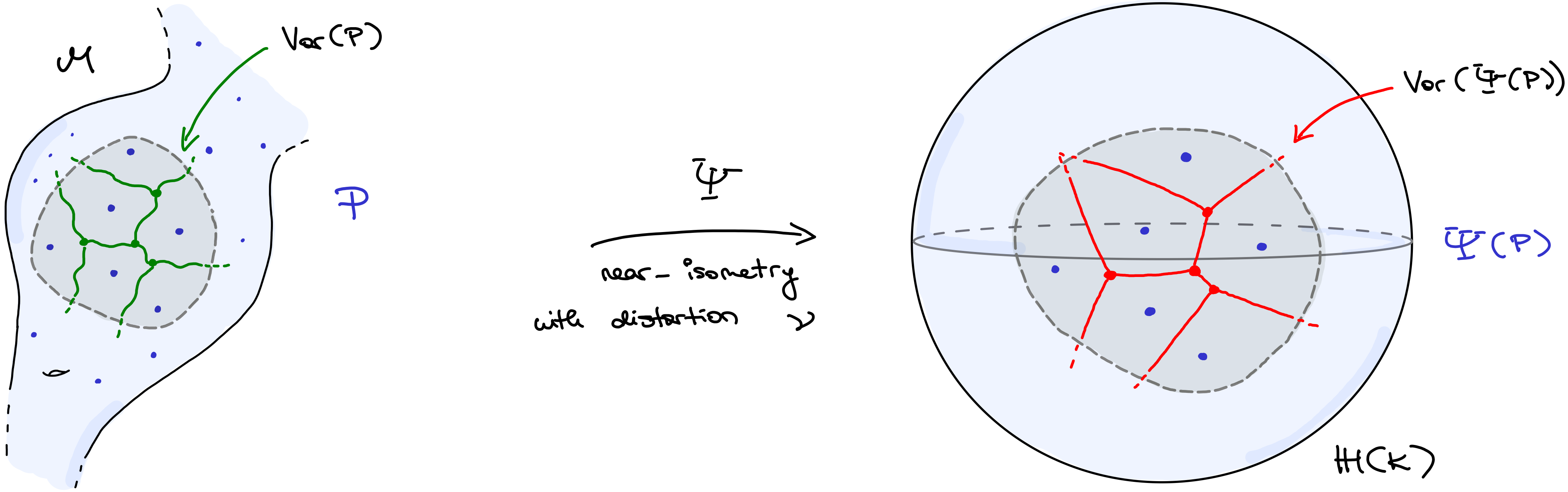
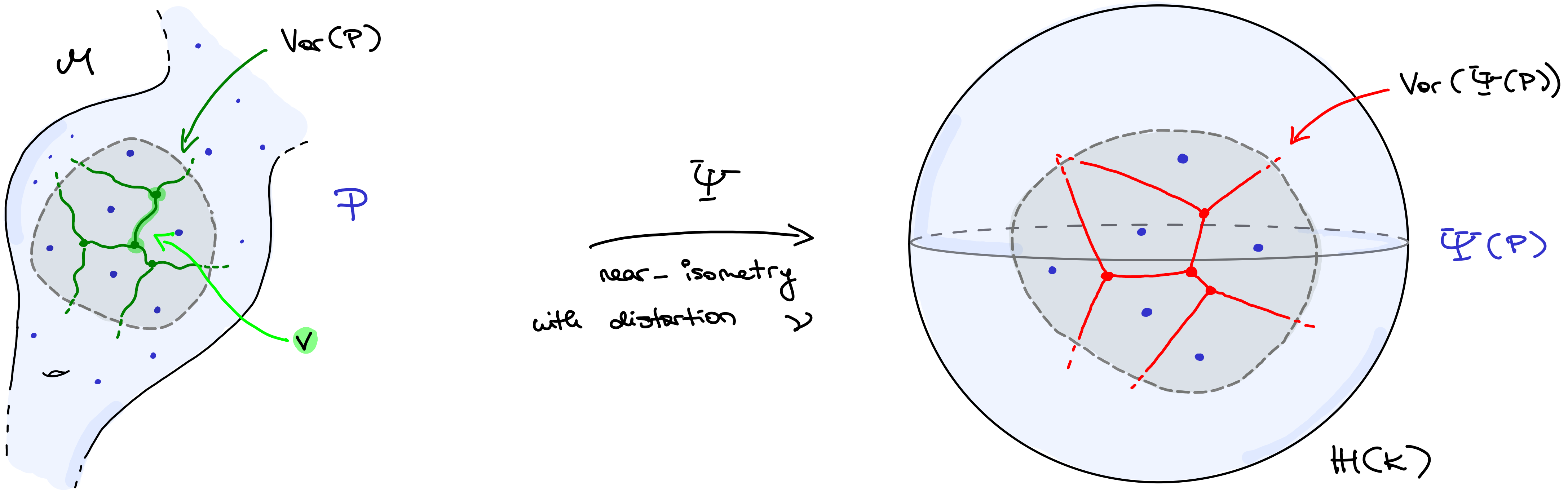
$$\Rightarrow \qquad \Psi(\quad) \subseteq \mathcal{L}_\gamma(\Psi(p_1),\Psi(p_2))$$



• use Vor(Ψ(P)) to control Vor(P) :



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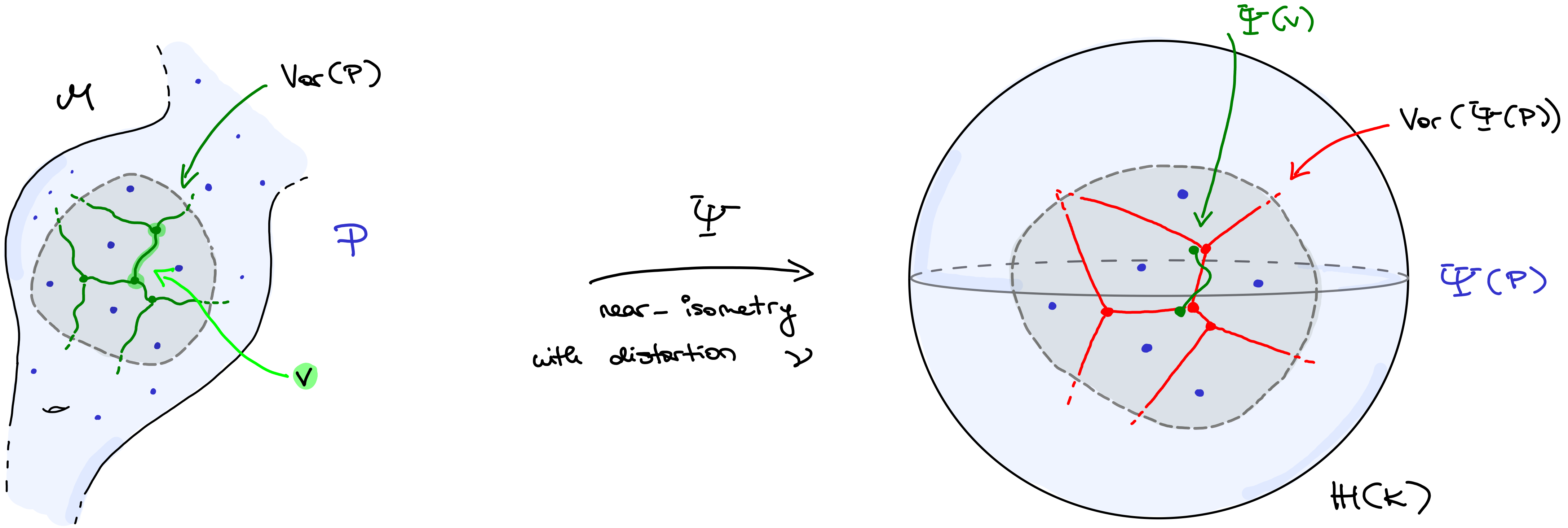
  - for each cell $v \in$ Vor(P),



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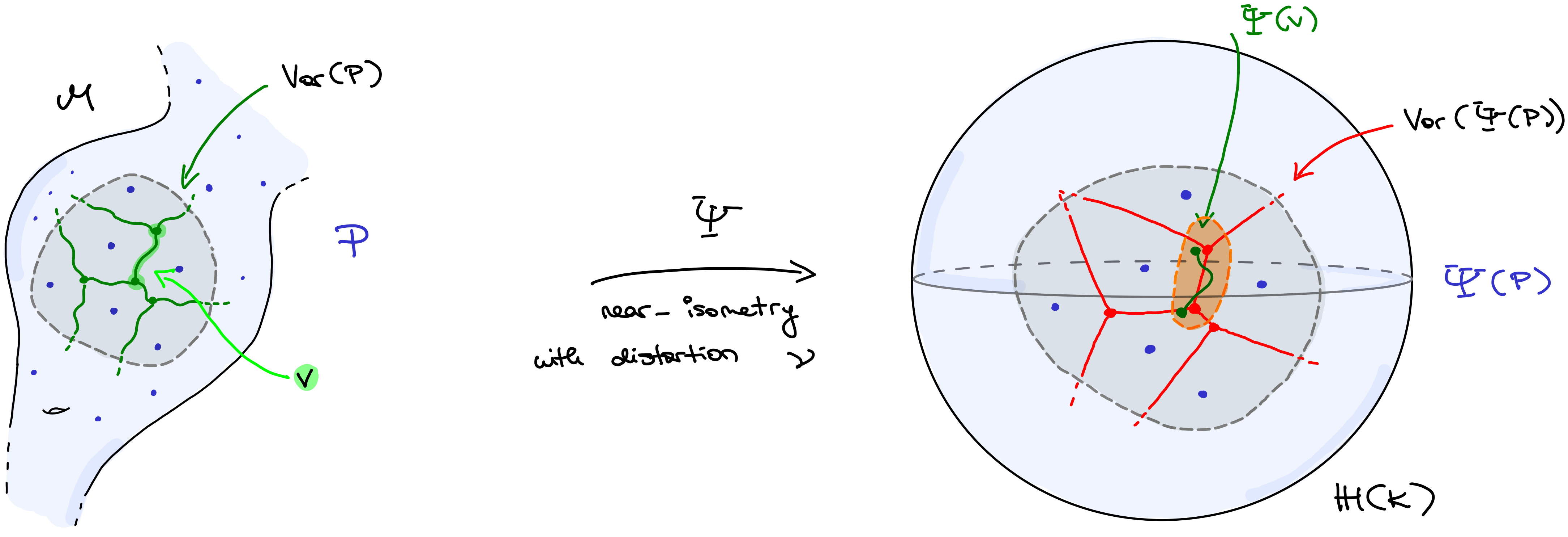

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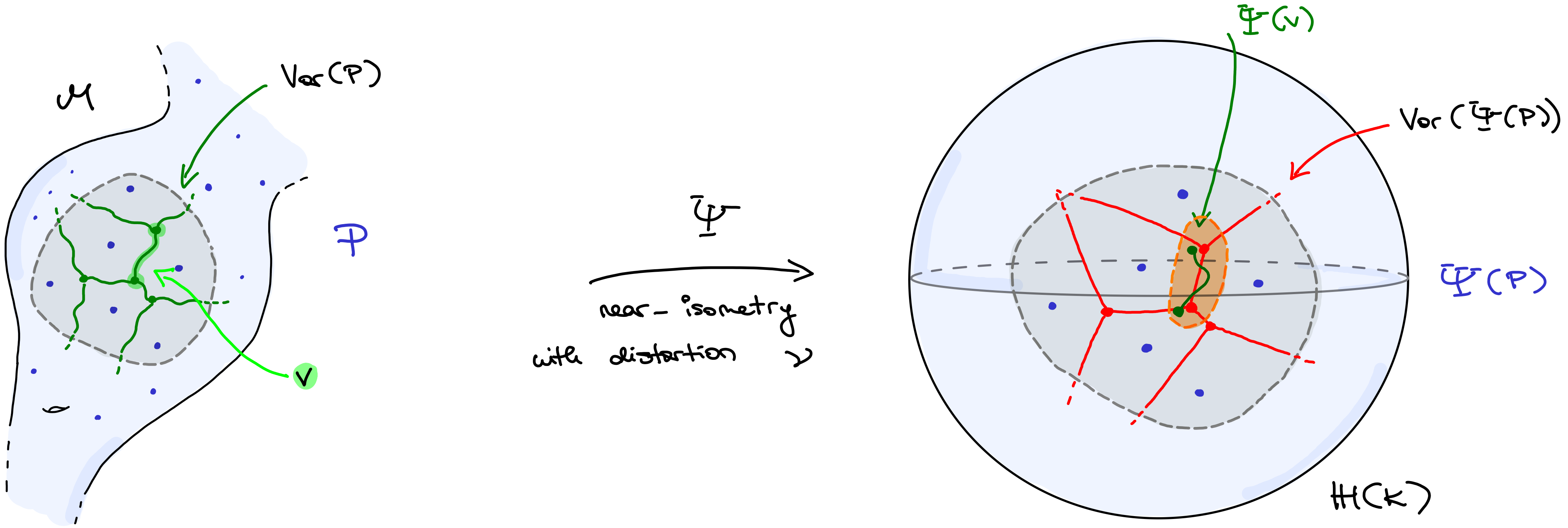

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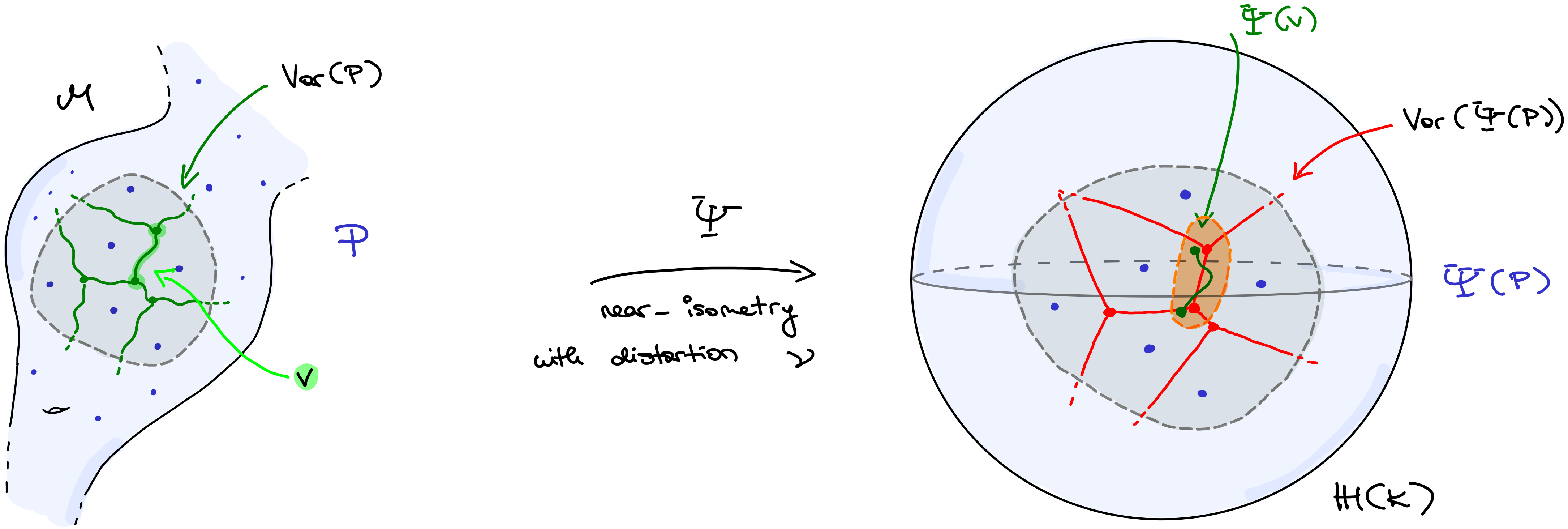

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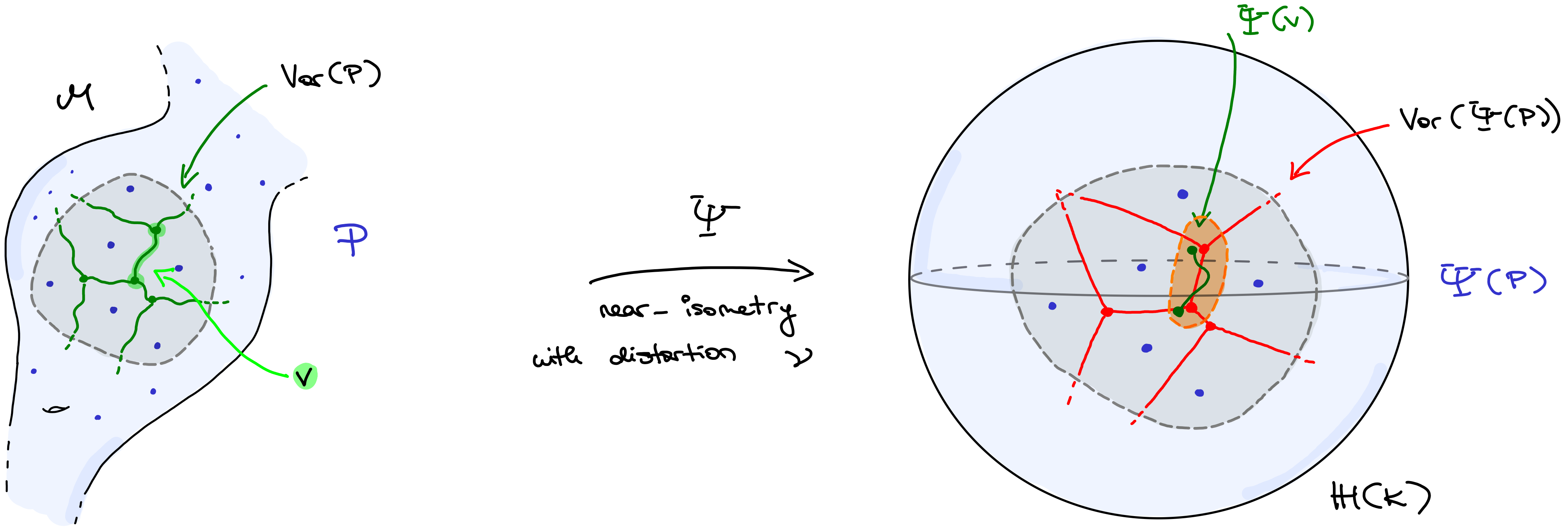
  - for $\nu \simeq 0$, $\approx$ $\in$ Vor(Ψ(P))



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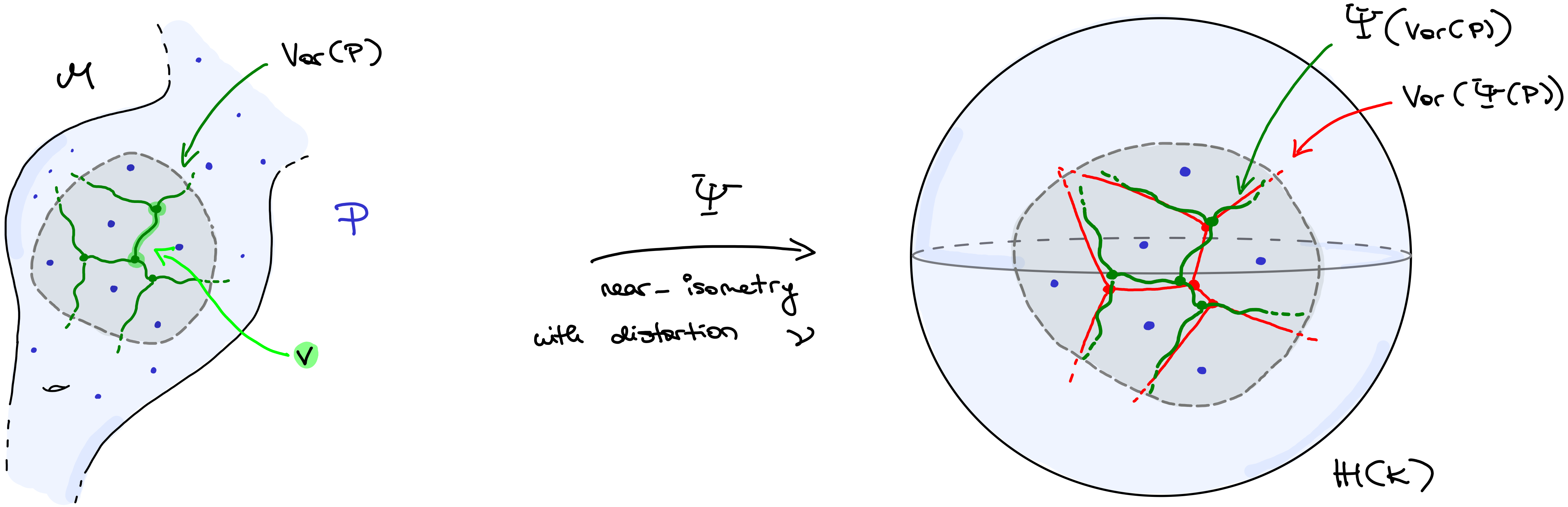
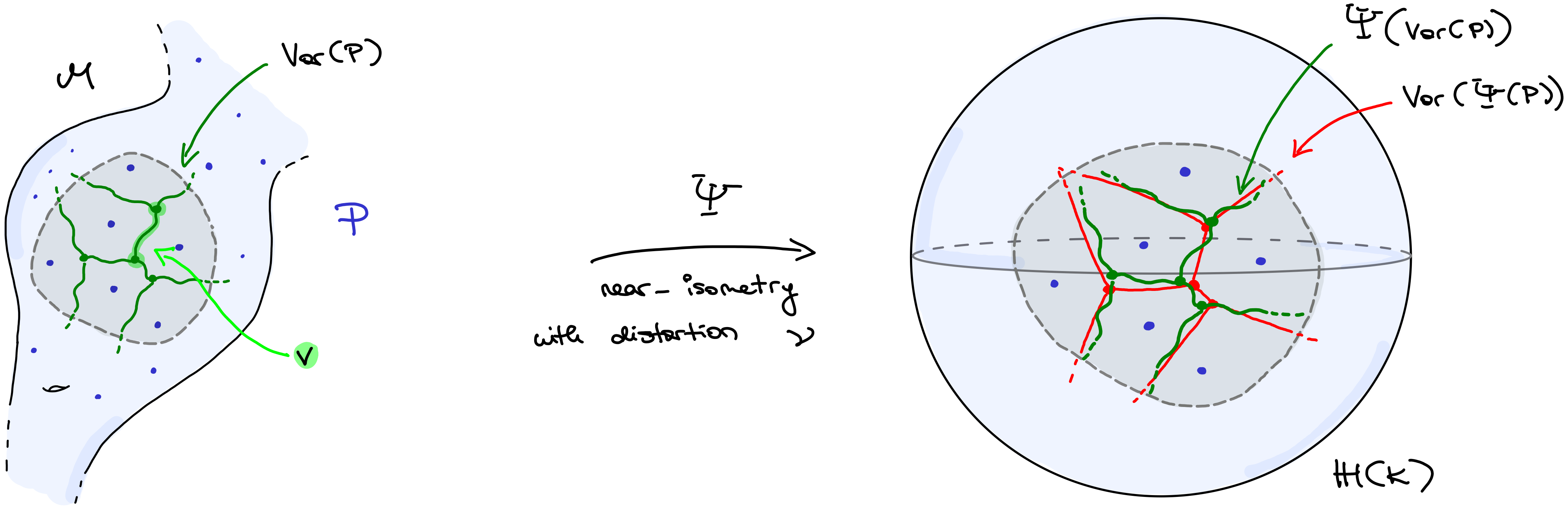
  - for $\nu \simeq 0$, $\approx$ $\in$ Vor(Ψ(P))

$\Rightarrow$ for small enough ν, Vor(Ψ(P)) and Vor(P) have the same combinatorics!



... and Vor(Ψ(P)) and Ψ(Vor(P)) are geometrically close



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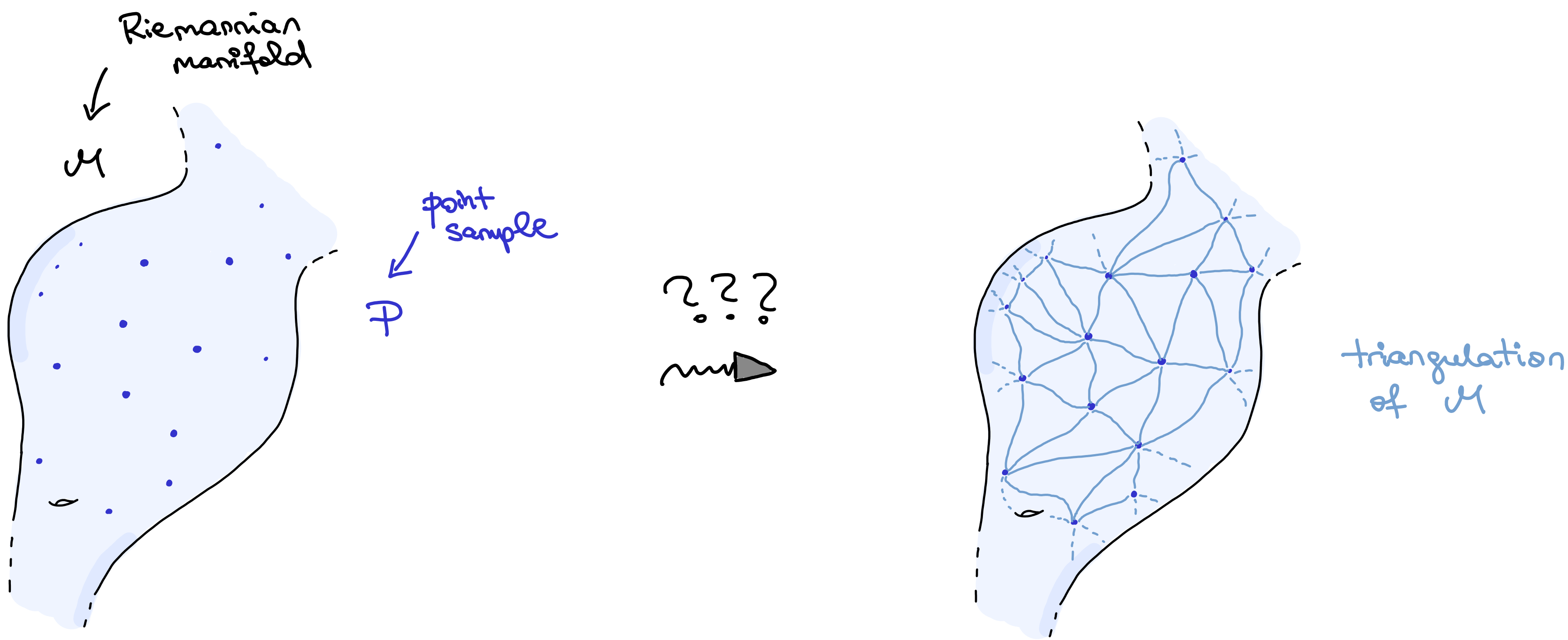
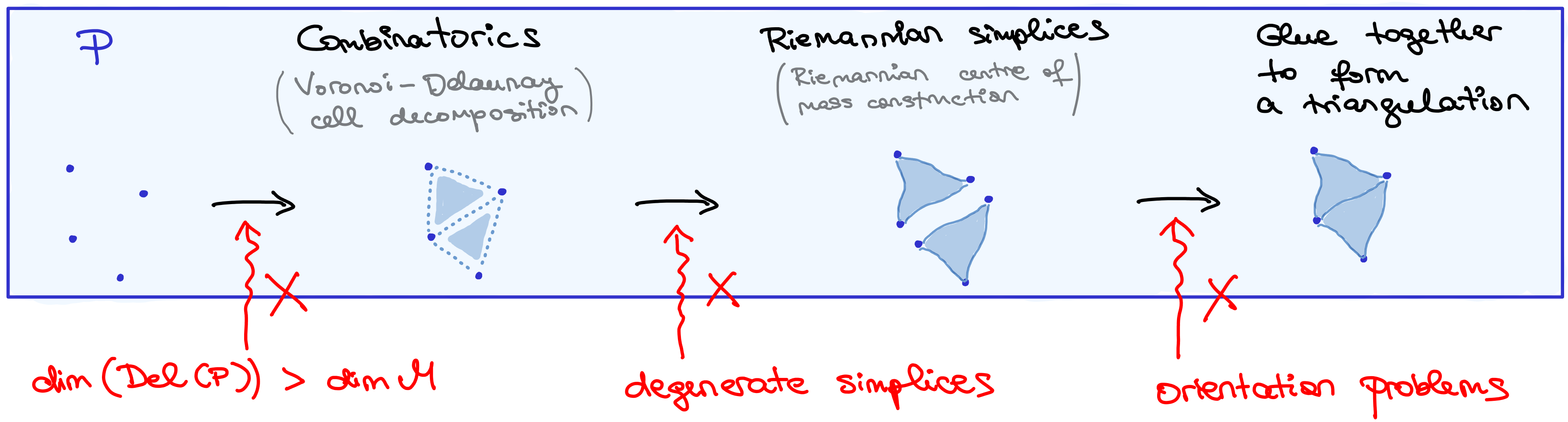
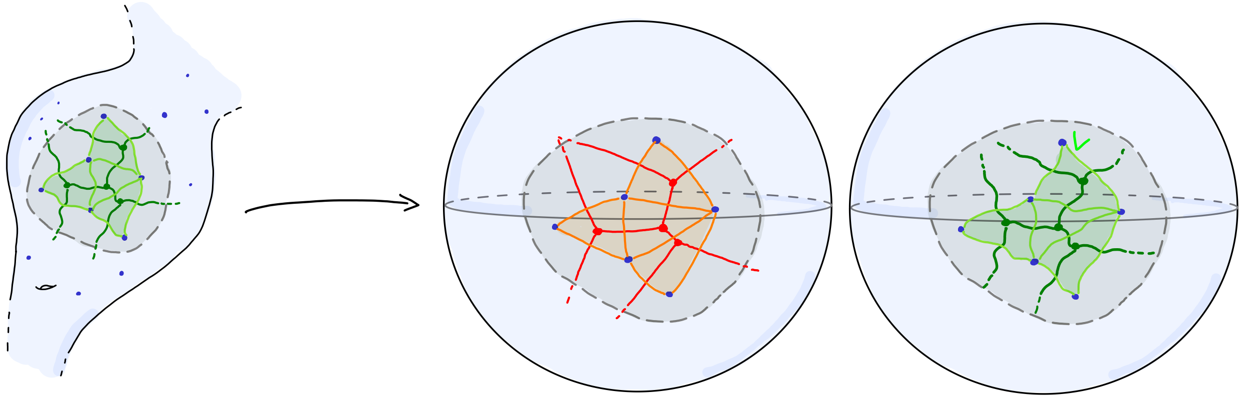
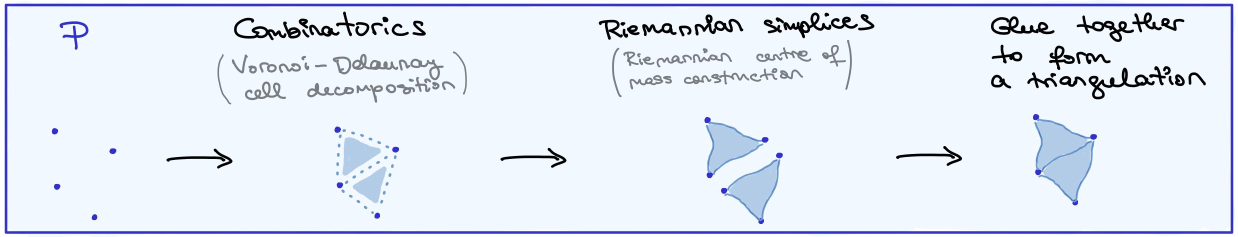
$\Rightarrow$ Del(Ψ(P)) and Ψ(Del(P)) are geometrically close

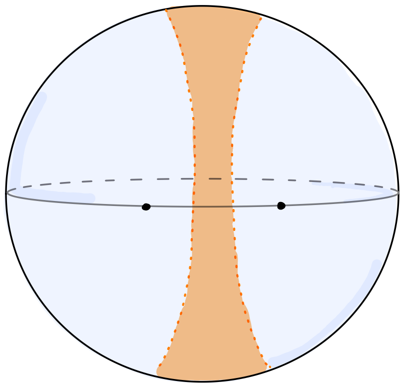


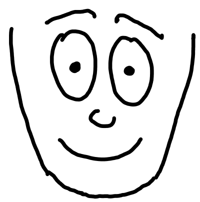
FAULTY PIPELINE







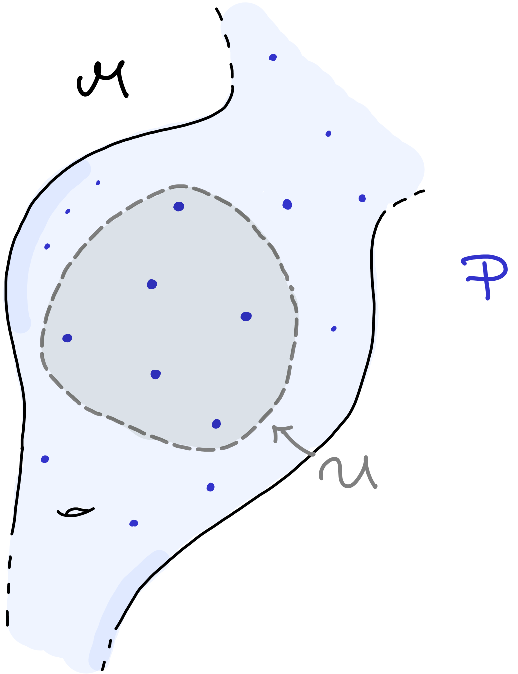




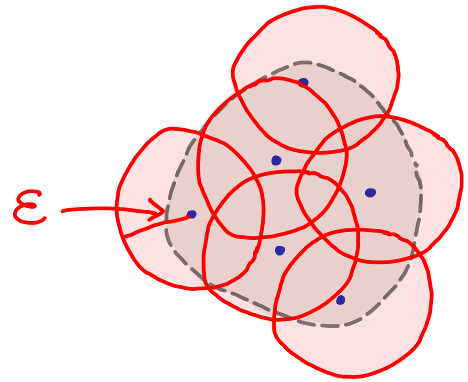
THANK YOU FOR YOUR ATTENTION!

Conditions on $\mathcal{P}$

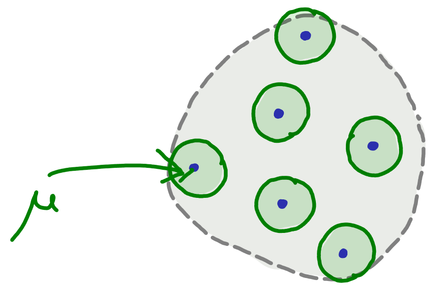
- local



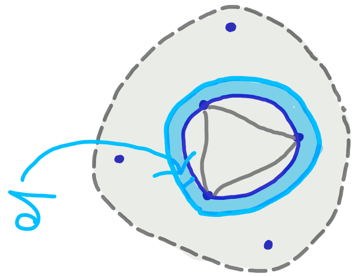
① Density:



② Sparsity:



③ Protection:



We bound $\varepsilon, \mu,$ and δ in terms of

the difference between the minimum and the maximum sectional curvature of $\mathcal{M}$ $(\mathcal{U})$.

⤳ in particular, $\delta \approx 0$ if curvature $\approx$ const.