# Embeddability of Graphs into 2-Dimensional Simplicial Complexes

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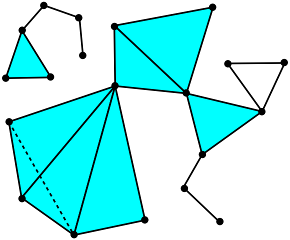
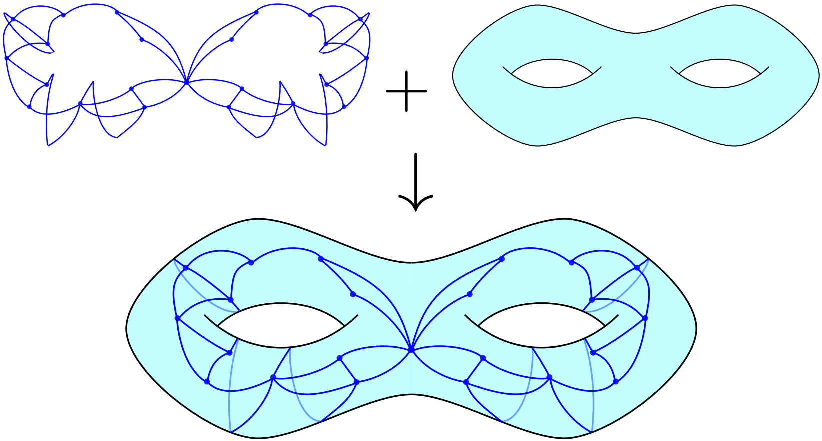
Joint work with Thomas Magnard



# Embedding graphs on surfaces

- Input: A graph $G$ with $n$ vertices and edges; a surface $S$ specified by its genus $g$ and its orientability
- Question: Decide whether $G$ has a topological embedding (a crossing-free drawing) into $S$.



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## Existing results

- [Thomassen, 1989]: NP-hard (when $g$ is part of the input)
- [Mohar, 1999]: $f(g) \cdot n$ (very technical)
- [Kawarabayashi et al., 2008]: $2^{\mathrm{poly}(g)} \cdot n$ (only appeared in extended abstract)
- Graph minor theory: $f(g) \cdot n^3$ [Robertson and Seymour, 1995]+[Adler et al., 2008].

# Our goal: beyond surfaces

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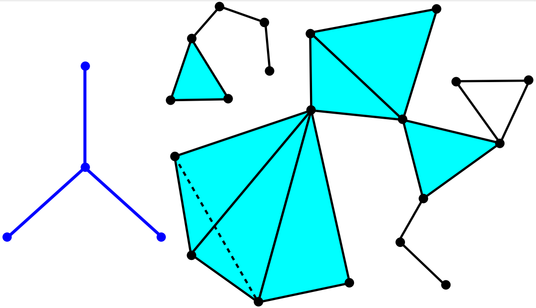

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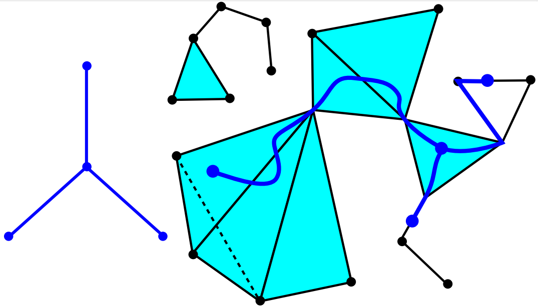

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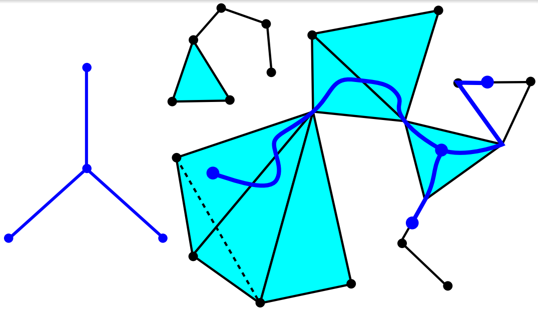

- everything is topological: no constraint on the embedding;
- actually, “2-dimensional” is an unnecessary restriction;
- NP-hard (surfaces are 2-complexes);
- the set of graphs embeddable on $C$ is not minor-closed;
- encompasses other known problems, e.g., crossing number.

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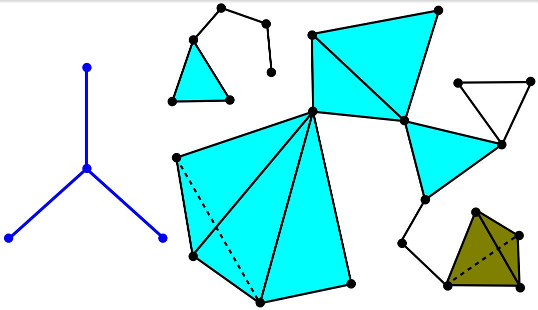

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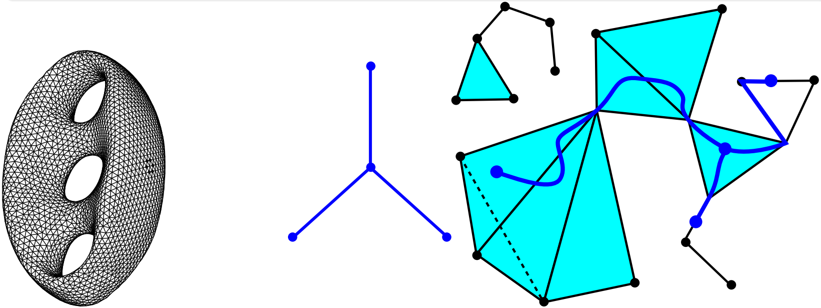

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An algorithm with running time $2^{\mathrm{poly}(c)} \cdot n^2$ where

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- Our algorithm is independent from the existing algorithms for surfaces, and simpler. . .
- but quadratic in $n$ instead of linear.
- Main strategy of the algorithm:
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**THANKS FOR YOUR ATTENTION!**