Finding weakly simple closed quasigeodesics on polyhedral spheres

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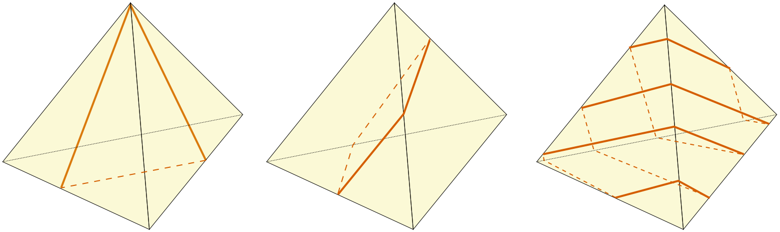
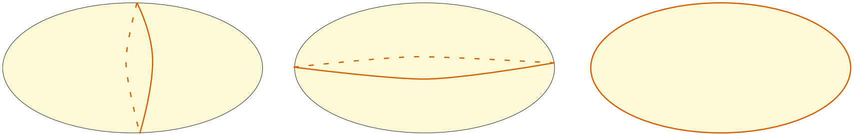


Joint work with Jean Chartier (LAMA, UPEC).

Geodesics on spheres

Theorem (Lyurstenik-Schnirrelman '29, etc.)

Any Riemannian sphere admits at least three simple closed geodesics.



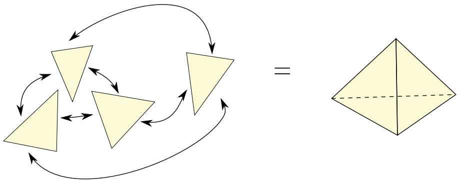
Main question

Cook up an *algorithm* to find those in a *discrete* setting.

What is ...?

What is a discrete sphere?

A *polyhedral sphere* is a sphere made of Euclidean polygons glued to each other.



Example: (Convex) polyhedra

What is a quasigeodesic?

A *quasigeodesic* is a curve that goes straight inside and between two polygons and forms equal angles when crossing a vertex.

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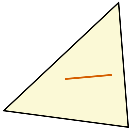

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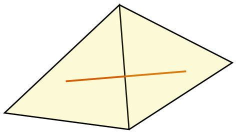

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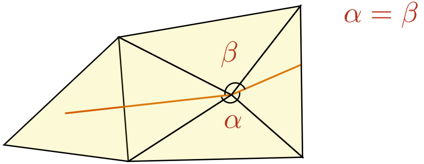

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Example: (Convex) polyhedra

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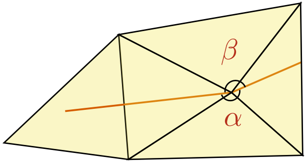
A *quasigeodesic* is a curve that goes straight inside and between two polygons and forms ~~equal angles~~ angles at *most*/least π when crossing a *convex*/concave vertex.



$\cancel{\alpha = \beta}$

$\alpha + \beta \leq 2\pi$

$\alpha \leq \pi$

$\beta \leq \pi$

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Theorem (Pogorelov '49)

Any convex polyhedron admits at least three simple closed quasigeodesics.

Results and questions

Question [Demaine, O'Rourke, Wyman '90-'07]

Cook up an algorithm to find those.

Theorem (Chartier, dM 2022)

Any polyhedral sphere admits a weakly simple closed quasigeodesic which crosses or uses $O(dM/h)$ times the edges of the sphere.

Corollary

Algorithm to find such a weakly simple closed quasigeodesic in exponential time.

Main technical tool: discrete version of a curve-shortening flow, adapted from the *disk flow* of [Hass and Scott '94].

Open question

Does there always exists a weakly simple closed quasigeodesic that crosses/uses each edge at most C times for some small constant C?